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Research Article

Optimizing ESG-Constrained Mean-Variance Portfolio using Spiral Optimization Algorithm

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Abstract
This research investigates ESG-constrained mean–variance portfolio optimization by incorporating buy-in threshold constraints using data from the Indonesian stock market. The optimization problem is formulated as a mixed-integer nonlinear programming (MINLP) model, which captures both the discrete investment decisions and the nonlinear nature of risk-return trade-offs. To solve this complex model, the spiral optimization algorithm (SOA) is employed due to its flexibility and efficiency in handling constrained optimization problems. The performance of SOA is benchmarked against other well-known metaheuristic algorithms, namely Particle Swarm Optimization (PSO) and Grey Wolf Optimizer (GWO), using portfolios consisting of 5 and 10 ESG-compliant stocks. Based on the empirical results for 5 stocks, we show that SOA gives the same results with PSO and GWO results. Meanwhile for 10 stocks, SOA gives consistent results than the results of PSO and GWO. Therefore, we conclude that SOA can be used in small number of stocks or extended stocks in solving this ESG-portfolio problems.

Keywords: buy-in threshold, ESG, portfolio optimization, spiral optimization algorithm

1. INTRODUCTION

Sustainability has gained importance due to social and economic inequality, deterioration of the environment, and climate change. Such influences have affected the administration of finances, which in previous decades only focused on profitability [1][2]. Investors, regulators, and other entities have started embracing green finance. This form of finance goes beyond the conventional investment and return risk analysis by disbursing funds based on Environmental Social and Governance (ESG) criteria [3]. This transformative shift entails the creation of models that configure investment portfolios and integrate unsustainability into the very center of finance.

The mean–variance portfolio optimization model, introduced by Markowitz [4], has served as the foundation of modern portfolio theory. It aims to allocate asset weights to minimize risk for a given level of expected return. However, traditional mean–variance models do not account for sustainability-related factors, nor do they address practical constraints such as minimum investment thresholds and non-linear compliance requirements like minimum ESG scores. There is research extending the classical model by embedding ESG scores objectives to make portfolio models more relevant for the green economy [5]. They used ESG in the objective function and they solve it using genetic algorithm. It is different with our research because we use ESG as the constraints not as the objective function. Then, we solve it using spiral optimization algorithm (SOA) as one of metaheuristic algorithms.

There are several research projects that have applied metaheuristic into constrained optimization problem, including power systems [6], engineering design [7], and multi-objective swarm intelligence algorithms to solve portfolio optimization [8]. Besides that, other scientists do descriptive statistics and OLS regression to optimize their sustainable portfolio [9]. There is research incorporated ESG factors into the classical mean–variance framework, demonstrating that sustainability constraints can be systematically embedded within traditional portfolio optimization models [10]. Furthermore, there is also research which analyzed ESG-oriented portfolios using two distinct approaches, namely the Markowitz Model (MM) and the Index Model (IM), and the reported a positive correlation between ESG scores and average stock returns, suggesting that higher sustainability performance does not

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Table 1. Sector classification and ESG score for 5 selected company.

Stock Code	Company Name	Sector	ESG Score
BBCA	Bank Central Asia	Finance/Banking	0.20
SMGR	Semen Indonesia	Construction Materials	0.40
DSNG	Dharma Satya Nusantara	Agribusiness/Plantations	0.60
TLKM	Telkom Indonesia	Telecommunications	0.80
UNVR	Unilever Indonesia	Consumer Goods	0.50

necessarily compromise financial profitability [11].

Motivated by these findings, as well as the earlier work of Febrianti et al. [12] on portfolio optimization using the Markowitz model, this research aims to extend the conventional mean–variance portfolio framework by explicitly incorporating ESG factors as optimization constraints. To effectively solve the resulting nonlinear and constrained optimization problem, this research employs the SOA, a metaheuristic optimization technique known for its strong global search capability and robustness in handling complex, high-dimensional financial optimization problems. Through this approach, the proposed model seeks to construct ESG-constrained optimal portfolios that achieve a balanced trade-off between risk minimization, return maximization, and sustainability performance.

Then, we choose SOA as the solution method in this research because it is strongly supported by its solid theoretical foundation and proven effectiveness in solving complex optimization problems. Tamura and Yasuda [13] demonstrated the convergence properties of SOA, particularly for optimization problems with low-dimensional search spaces, providing early theoretical validation of the algorithm's stability and reliability [13]. Subsequentlye, Kania and Sidarto [14] extended the applicability of SOA by showing its strong capability in solving mixed-integer nonlinear programming (MINLP) problems, highlighting the algorithm's effectiveness in handling nonlinear objective functions and discrete decision variables under multiple constraints. This feature is particularly relevant for portfolio optimization problems that incorporate practical investment constraints such as budget limits, ESG thresholds, and asset selection rules. Previous work [15] also concern with portfolio with constraint so that it becomes nonlinear portfolio. Furthermore, Febrianti et al. [12] developed an enhanced SOA framework that ensures robust convergence across all dimensional settings, including high-dimensional optimization problems. This development significantly expands the applicability of SOA to large-scale financial optimization problems involving a substantial number of assets.

We choose SOA instead of grey wolf optimizer (GWO) or particle swarm optimization (PSO) due to its strong theoretical convergence properties, structural simplicity, and proven effectiveness in solving MINLP problems, which are central to ESG -constrained portfolio optimization. We also compare our portfolio results with GWO and PSO. This paper makes three main contributions. It proposes a constrained mean–variance portfolio optimization model that integrates ESG scores and buy-in threshold constraints into a unified framework. The resulting optimization problem is solved using the SOA and evaluated through numerical simulations based on real financial data, demonstrating the algorithm's capability in handling nonlinear and constrained portfolio optimization problems. The novelty of this research lies in the development and empirical evaluation of a unified ESG-constrained mean–variance portfolio optimization framework that explicitly incorporates both KESGI-based ESG thresholds and buy-in constraints within a MINLP formulation. Then, we change the portfolio problem into an unconstrained optimization problem by using penalty function.

This paper consists of four sections. The first section defines introduction. In Section 2, we explain about the materials and methods. In Section 3, we show the results of our SOA and comparison results with PSO and GWO. The last section is conclusions.

2. MATERIALS AND METHODS

This section focuses on the appropriate ideas and literature that enable this investigation. The method begins with the identification of the latest trends in ESG-based portfolio optimization models, subsequently addressing additional portfolio optimization constraints such as ESG investment limits and buy-in restrictions. We also show about SOA.

2.1. ESG-Based Portfolio Optimization

The integration of ESG criteria into financial decision-making has gained substantial momentum, driven by growing regulatory pressure and investor demand for sustainable practices [16]. Traditional mean-variance (M–V) models, while foundational in portfolio theory, are insufficient to address sustainability preferences without modification.

To align with ESG mandates, scholars have extended the M–V framework by either maximizing ESG scores as a third objective, incorporating ESG as an additive constraint, or setting minimum thresholds for ESG compliance [17]. These enhancements have demonstrated the ability to generate portfolios that are more resilient to long-term systemic risks and better aligned with environmental and sustainability objectives. However, they also introduce new challenges, such as reduced diversification and increased model complexity.

The implementation of ESG practices by companies helps them explore opportunities for sustainability and economic growth. In 2024, Katadata expanded its ESG Index assessment to cover eight industrial sectors: financial/banking, mining, plantation, food and beverage, transportation & logistics, chemicals, energy, and hospitality. The ESG Index assessment is based on the Katadata ESG Index report as of June 2024 [18]. The mathematical formula for calculating the actual score of a specific category for a company is given as Equation (1);

$$Indicator\ Score_i = \left(\frac{X_i - X_{min}}{X_{max} - X_{min}}\right) \times 100 \quad (1)$$

where X_i is equals to actual value of indikator i, X_{min} is minimum values of indicator across all companies being compared and X_{max} is maximum values of indicator across all companies being compared. Then, we also use reverse indicator as a smaller value indicates better performance. We use the formula as in Equation (2).

$$Reverse\ Indicator\ Score_i = 100 - \left[\left(\frac{X_i - X_{min}}{X_{max} - X_{min}}\right) \times 100\right] \quad (2)$$

Then, we use ESG score based on Katadata ESG Index (KESGI) [18]. There are two types of KESGI based on the sectors in the stocks field. First, based on sectors of mining, plantation, food & beverage, transportation & logistics, chemicals, energy, and hospitality, we use KESGI formula as follows Equation (3).

$$KESGI = 50\%\ Environment + 30\%\ Social + 20\%\ Governance \quad (3)$$

Table 2. Sector classification and ESG score for 10 selected company.

Stock Code	Company Name	Sector	ESG Score
BBCA	Bank Central Asia	Finance/Banking	0.80
BBRI	Bank Rakyat Indonesia	Finance/Banking	0.78
SMGR	Semen Indonesia	Construction Materials	0.60
DSNG	Dharma Satya Nusantara	Agribusiness/Plantation	0.72
TLKM	Telkom Indonesia	Telecommunications	0.74
UNVR	Unilever Indonesia	Consumer Goods	0.76
INDF	Indofood Sukses Makmur	Food and Beverages	0.67
KLBF	Kalbe Farma	Healthcare	0.70
TBIG	Tower Bersama Infrastructure	Infrastructure	0.65
EXCL	XL Axiata	Telecommunications	0.68

Table 3. Sector classification and ESG score for 15 selected company.

Stock Code	Company Name	Sector	ESG Score
BBCA	Bank Central Asia	Finance/Banking	0.80
BBRI	Bank Rakyat Indonesia	Finance/Banking	0.78
SMGR	Semen Indonesia	Construction Materials	0.60
DSNG	Dharma Satya Nusantara	Agribusiness/Plantation	0.72
TLKM	Telkom Indonesia	Telecommunications	0.74
UNVR	Unilever Indonesia	Consumer Goods	0.76
INDF	Indofood Sukses Makmur	Food and Beverages	0.67
KLBF	Kalbe Farma	Healthcare	0.70
TBIG	Tower Bersama Infrastructure	Infrastructure	0.65
EXCL	XL Axiata	Telecommunications	0.68
PGAS	Perusahaan Gas Negara	Energy Infrastructure	0.60
ANTM	Aneka Tambang	Mining	0.63
WIKA	Wijaya Karya	Construction	0.61
ICBP	Indofood CBP Sukses Makmur	Food and Beverages	0.66
BFIN	BFI Finance Indonesia	Finance/Leasing	0.64

Second, based on the final KESGI score for the financial/banking sector is computed as Equation (4).

$$KESGI = 20\%\ Environment + 30\%\ Green\ Economy + 30\%\ Social + 20\%\ Governance \quad (4)$$

In the context of ESG-constrained portfolio optimization, the sustainability performance of a portfolio is generally evaluated using an aggregate ESG score computed as a weighted sum of the individual asset scores. Specifically, the contribution of the i- th asset to the portfolio's ESG performance can be expressed as the product $s_i y_i$, where s_i denotes the ESG rating of the asset and y_i represents its portfolio weight. This formulation ensures that both the qualitative sustainability characteristics of an asset and the quantitative allocation decision are simultaneously reflected in the measurement of portfolio-level ESG compliance. The aggregate score can then be expressed as Equation (5);

$$S_p = \sum_i s_i y_i = \mathbf{s}^T \mathbf{y} \quad (5)$$

where each term $s_i y_i$ reveals the effective contribution of aset i. We analyze the decomposition of $s^T y$ because it is essential and providing transparency in portfolio construction. We can detect concentration of sustainability exposure, and clarifies the trade-off between financial performance and ESG compliance. Consequently, the inclusion of weighted ESG contributions is critical in ESG-constrained optimization models, as it allows for both the enforcement of minimum thresholds and the assessment of asset-level sustainability influence.

The computed KESGI scores for each company, as determined using Equations (3) – (4), represent a composite ESG performance metric that combines ESG and green economy dimensions. Once calculated, these scores form the basis for the ESG vector $s \in \mathbb{R}^n$ denotes the matrix of ESG scores for each asset. Then, each element $s_i \in s$ corresponds to the KESGI score of assets i. Let $s^T \in \mathbb{R}^n$ denote the transpose matrix of ESG scores for each asset and let S_{min} denote the minimum acceptable ESG score for the portfolio. In the portfolio optimization model, the ESG constraint is formulated as Equation (6).

$$\mathbf{s}^T \mathbf{y} \geq S_{min} \quad (6)$$

This condition ensures that the constructed portfolio meets a minimum standard for sustainability, thereby aligning investment decisions with responsible investing principles. In mathematical terms, the constraint in Equation (6) guarantees that the weighted average ESG score of the selected portfolio equals or exceeds a predetermined sustainability threshold. By directly linking KESGI-based ESG scores to the optimization model, sector-specific sustainability performance is explicitly captured, while computational tractability is preserved due to the linear nature of the constraint.

The novelty of this study lies in the integration of KESGI-derived ESG scores with buy-in threshold constraints into a mixed-integer, non-convex portfolio optimization framework. This hybrid formulation reflects real-world investment conditions more accurately than classical models, yet also increases computational complexity. To solve such a problem efficiently, we employ a derivative-free SOA enhanced with a penalty function mechanism, enabling the algorithm to handle both continuous and discrete decision variables while maintaining robust search performance in non-convex optimization spaces.

2.2. Portfolio Constraints: ESG and Buy-in Thresholds

Practical portfolio construction inevitably involves addressing multiple real-world constraints. Two particularly important ones are: (i) buy-in thresholds, which ensure that each selected asset receives a meaningful allocation (e.g., ≥ 5%), preventing the inclusion of negligible weights that are impractical or uneconomical [19], (ii) minimum ESG scores, where the total ESG-weighted portfolio score must exceed a specified threshold to qualify as "sustainable". These constraints, when combined with expected return and budget constraints, produce a MINLP problem. The model includes both continuous variables (portfolio weights) and binary decision variables (buy-in), making the feasible region non-convex and significantly more difficult to solve using classical optimization approaches [20].

2.3. Mathematical Formulation for ESG Portfolio

Let portofolio weights $y = (y_1, y_2, \ldots, y_n)^T \in \mathbb{R}^n$, binary indicator for buy-in threshold $z = (z_1, z_2, \ldots, z_n)^T \in \{0,1\}^n$, expected return vector $r = (r_1, r_2, \ldots, r_n)^T$, minimum return target R_p, covariance matrix Q (adjusted to be positive semi-definite via $Q + \lambda I$), ESG score vector s, minimum ESG threshold S_{min}, l_i minimum allocation for asset i, $u_i = 1$ upper bound for full allocation.

Referring to the formulation of the Mixed Integer Nonlinear Programming (MINLP) problem can be expressed as follows as Equation (7) [12] [14],

$$\min_{\mathbf{x} \in \mathbb{R}^n} f(\mathbf{x}) \tag{7}$$

subject to

$gi\,(\mathrm{x}) = 0,\ i = 1, 2, \ldots, M$

$h_j\,(\mathrm{x}) \le 0,\ j = 1, 2, \ldots, N$

$\mathrm{x} = \{x_1, x_2, \ldots, x_q, x_{q+1}, \ldots, x_n\}$

Where $x_1, x_2, \ldots, x_q$ represent integer-valued variables for a given integer q. This constrained optimization problem can be transformed into an unconstrained form using a penalty function approach. Accordingly, the objective becomes the minimization of a penalty-based function defined as Equation (8).

$$F(\mathbf{x}, \alpha_i, \beta_j) = f(\mathbf{x}) + \sum_{i=1}^{M} \alpha_i g_i^2(\mathbf{x}) + \sum_{j=1}^{N} \beta_j \left(\max\left(h_j(\mathbf{x}), 0\right)\right)^2 \tag{8}$$

In this formulation, α_i and β_j denote large positive constants acting as penalty parameters. For simplicity, we assume uniform penalty coefficients,

Table 4. Spiral optimization algorithm parameters.

Parameter	Value
Maximum iteration	1000
Rotation angle	π / 4
The number of search points (m)	100
The spiral radius (r)	0.99

Table 5. SOA, PSO, and GWO for five stocks ESG portfolio optimization.

Stocks Code	SOA			PSO			GWO		
	y_i	z_i	Risk	y_i	z_i	Risk	y_i	z_i	Risk
BBCA	0.0500	1		0.0500	1		0.0500	1	
SMGR	0.1871	1		0.1871	1		0.1871	1	
DSNG	0.6630	1	0.8827	0.6630	1	0.8827	0.6630	1	0.8827
TLKM	0.0500	1		0.0500	1		0.0500	1	
UNVR	0.0500	1		0.0500	1		0.0500	1	

i.e., $\alpha_i = \alpha$, for all $i = 1, 2, \ldots, M$ and $\beta_i = \beta$, for all $i = 1, 2, \ldots, N$.

The integration of ESG and buy-in threshold constraints into the classical mean–variance framework transforms the problem from a convex quadratic programming formulation into a MINLP problem. This transformation arises due to three main factors. First, the introduction of binary decision variables $z_i \in \{0,1\}$ to represent the buy-in threshold constraint leads to a mixed-integer decision space. Second, the combination of ESG and buy-in threshold constraints with the quadratic objective function to minimize risk denoted as $y^T Q y$ induces non-convexity in the feasible region, thereby eliminating the guarantee that a local minimum is also a global minimum. Third, the coupling of continuous portfolio weights y_i with binary selection variables z_i generates combinatorial complexity, which significantly increases the computational difficulty of the optimization process. Consequently, the resulting problem can be expressed in the general MINLP problem as in [12]. Therefore, we show mean-variance portfolio with ESG and buy-in thresholds constraints formula as in Equation (3) as follows Equation (9).

$$\min_{y,z} \mathbf{y}^T \mathbf{Q} \mathbf{y} \tag{9}$$

subject to

$$\bar{r}^T y = R_p$$
$$e^T y = 1$$
$$s^T y \geq S_{min}$$

$$l_i z_i \leq y_i \leq u_i z_i, \qquad \forall i = 1, 2, \ldots, n$$
$$z_i \in \{0,1\}, \qquad y_i \geq 0, \qquad \forall i = 1, 2, \ldots, n$$

However, directly solving the resulting MINLP is computationally challenging due to its non-convex feasible region, mixed-integer nature, and the presence of multiple interdependent constraints. To efficiently handle this complexity, a penalty function approach is adopted, transforming the constrained problem into an unconstrained one by incorporating constraint violations into the objective function. This method allows the optimization process, particularly when using metaheuristic algorithms such as SOA, to explore a broader search space while progressively guiding solutions toward feasibility. The penalty parameters control the severity of constraint violations, ensuring that feasible solutions are favored and the algorithm converges toward the global optimum.

2.4. Penalty Function Framework

To handle the complex constraints embedded in the green portfolio optimization model, namely return targets, ESG score thresholds, full investment conditions, and buy-in rules, a penalty function approach is employed. This transformation converts the constrained optimization problem into an unconstrained one, allowing both metaheuristic and deterministic methods to operate efficiently in the feasible search space.

The constrained objective function, which is constructed using Equation (8) from the problem in Equation (7), is defined as Equation (10);

$$F(y,z) = \mathbf{y}^T \mathbf{Q} \mathbf{y} + \rho(\mathbf{e}^T\mathbf{y} - 1)^2 + \mu\left(\bar{\mathbf{r}}^T\mathbf{y} - R_p\right)^2 + \gamma(\max(0, S_{min} - \mathbf{s}^T\mathbf{y}))^2 + \sum_{i=1}^{n} \alpha_i(\max(0, l_i z_i - y_i))^2 + \sum_{i=1}^{n} \beta_i(\max(0, y_i - u_i z_i))^2 \tag{10}$$

where $\rho, \mu, \gamma, \alpha_i, \beta_i > 0$ are positive penalty parameters that determine the magnitude of the penalty applied for violating each constraint. The first term, $y^T Q y$, represents the portfolio variance, which serves as the primary objective to be minimized. The second term, $\rho(e^T y - 1)^2$, penalizes deviations from the budget constraint, ensuring that the sum of asset weights equals one. The third term, $\mu(\bar{r}^T y - R_p)^2$, penalizes deviations from the target portfolio return. The fourth term, $\gamma[\max(0, s^T y - S_{min})]^2$, enforces the ESG constraint by imposing a penalty whenever the portfolio's weighted ESG score falls below the specified minimum threshold. The last two summation terms enforce the buy-in threshold constraints: the α_i term penalizes violations where the allocated weight is less than the lower bound when an asset is selected, and the β_i term penalizes violations where the allocated weight exceeds the upper bound. The last two terms enforce the buy-in constraints through logical consistency between y_i and z_i.

By calibrating the penalty parameters to appropriate magnitudes, the optimization process imposes substantial costs on infeasible solutions while retaining the capacity to explore diverse regions of the search space. This balance between constraint enforcement and exploratory capability is critical in solving non-convex, mixed-integer portfolio optimization problems, where feasible regions are often fragmented and irregular. This formulation allows algorithms like SOA to search the solution space smoothly, while progressively steering the solution toward feasibility. The penalty parameters are tuned empirically to balance between solution quality and constraint satisfaction.

2.5. Spiral Optimization Algorithm

The SOA, introduced by Tamura and Yasuda [13], is a population-based metaheuristic inspired by the geometric behavior of spirals. SOA operates by generating new candidate solutions through controlled spiral trajectories around elite individuals in the population [12]. Each iteration applies the transformation (Equation (11));

$$\mathbf{x}_{k+1} = \mathbf{x}_k + r_k R(\theta)(\mathbf{x}^* - \mathbf{x}_k) \quad (11)$$

where x* denotes the current best solution vector, $r_k = r_0\delta^k$ is the contraction factor controlling the spiral radius, with initial radius $r_0 > 0$ and decay rate $0 < \delta < 1$, then $R(\theta)$ is a rotation matrix parameterized by angle θ, ensuring diversified search directions.

The spiral mechanism balances global exploration and local exploitation, helping the algorithm escape local optima while converging smoothly toward global solutions. Unlike traditional metaheuristics that rely on random dispersion (e.g., PSO), SOA's geometric progression enables more structured convergence and better handling of constraint-dense problems, including those found in sustainable portfolio construction [18]. Therefore, we describe SOA for solving ESG-Constrained Portfolio Optimization as in Algorithm 1. This tailored integration of the SOA with the penalty function framework provides a computationally efficient and robust mechanism for identifying

Table 6. SOA results for ten stocks ESG portfolio.

Stocks Code	SOA								
	(Repeated 1)			(Repeated 2)			(Repeated 3)		
	y_i	z_i	Risk	y_i	z_i	Risk	y_i	z_i	Risk
BBCA	0.0500	1	0.00047	0.0500	1	0.00046	0.0505	1	0.00044
BBRI	0.0733	1		0.0568	1		0.0664	1	
SMGR	0.0500	1		0.0501	1		0.0504	1	
DSNG	0.1899	1		0.2549	1		0.2272	1	
TLKM	0.1310	1		0.0991	1		0.0711	1	
UNVR	0.0500	1		0.0500	1		0.0500	1	
INDF	0.1164	1		0.0530	1		0.0732	1	
KLBF	0.1950	1		0.1983	1		0.1741	1	
TBIG	0.0942	1		0.0754	1		0.1812	1	
EXCL	0.0501	1		0.1123	1		0.0561	1	

Table 7. PSO results for ten stocks ESG portfolio optimization.

Stocks Code	PSO								
	(Repeated 1)			(Repeated 2)			(Repeated 3)		
	y_i	z_i	Risk	y_i	z_i	Risk	y_i	z_i	Risk
BBCA	0.0500	1	0.00091	0.0000	0	0.00080	0.0000	0	0.00102
BBRI	0.0500	1		0.0722	1		0.0500	1	
SMGR	0.0000	0		0.0500	1		0.0000	0	
DSNG	0.6791	1		0.5776	1		0.7500	1	
TLKM	0.0000	0		0.0502	1		0.0500	1	
UNVR	0.0500	1		0.0500	1		0.0000	0	
INDF	0.0000	0		0.0500	1		0.0000	0	
KLBF	0.0709	1		0.0500	1		0.0500	1	
TBIG	0.0500	1		0.0500	1		0.0500	1	
EXCL	0.0500	1		0.0500	1		0.0561	1	

portfolios that minimize risk while simultaneously meeting financial return targets and satisfying stringent ESG and investment constraints within complex and non-convex search spaces.

SOA offers several advantages for ESG-constrained portfolio optimization compared to genetic algorithms (GA), particle swarm optimization (PSO), and differential evolution (DE). SOA employs a geometrically structured spiral search mechanism that enables smoother convergence and reduces oscillatory behavior commonly observed in PSO. Unlike GA and DE, which rely heavily on stochastic operators such as crossover and mutation, SOA maintains directional search toward elite solutions, improving convergence stability in constraint-dense and non-convex problems. These characteristics make SOA particularly suitable for mixed-integer portfolio optimization problems involving ESG and buy-in constraints. In this research, SOA is customized for portfolio optimization by encoding each candidate solution as a vector of continuous portfolio weights and binary buy-in variables. Constraint violations are handled through a penalty-based objective function, allowing SOA to explore infeasible regions while converging toward feasible solutions.

Algorithm 1. SOA for Solving ESG Portfolio Problems

1. Preliminaries: Spiral input parameter and portfolio ESG input parameter
2. Initialization
3. Generate initial z_j and y_j randomly in the feasible search space.
4. The penalty objective function $F(y,z)$ for solving ESG-Constrained Portfolio Optimization using SOA is stated as in Equation (10)
5. Find y* and z*
6. Update the value of y_j (k) to be y_j (k+1) and z_j (k) to be z_j (k+1)
7. Update the values of y* and z*
8. If $k = k_{max}$, end to the program. Otherwise, choose $k = k + 1$ and return to step 5
9. Choose $y^*=(y_1^* \; y_2^* \; \dots \; y_i^* \; \dots \; y_n^*)^T$ and $z^*=(z_1^* \; z_2^* \; \dots \; z_i^* \; \dots \; z_n^*)^T$
10. Ouput y* and z*

3. RESULTS AND DISCUSSIONS

This section presents the experimental setup and findings from the ESG-constrained portfolio optimization using SOA. The objective is to evaluate optimal portfolio weights under return, risk, and ESG constraints across three portfolio sizes: 5, 10, and 15 ESG-compliant Indonesian stocks. We also compare the SOA results with GWO and PSO results.

3.1. Data Description

This research obtained financial data from the IDX and ESG scores from the KESGI. It used daily closing prices from January 2021 to December 2024 to compute asset returns. The study calculated returns using logarithmic returns, defined as in Equation (12) [12];

$$r_{i,t} = ln \frac{P_{i,t}}{P_{i,t-1}} \quad (12)$$

Where $P_{i,t}$ denotes the closing price of asset i at time t. This research also constructed the variance–covariance matrix Q directly from the daily return series to represent the dependence structure among assets. The empirical dataset consists of ESG-screened companies listed on the IDX. Three portfolio sizes are analyzed: 5, 10, and 15 stocks, selected to represent diverse sectors relevant to the Indonesian economy, including finance, construction, agribusiness, telecommunications, consumer goods, energy, and infrastructure.

3.2. Five-Stock Portfolio

The dataset used in this research comprises five publicly listed companies on the IDX ESG Index that collectively represent a diverse range of sectors significant to the national economy and the ESG investment landscape. These companies are Bank Central Asia (BBCA), Semen Indonesia (SMGR), Dharma Satya Nusantara (DSNG), Telkom Indonesia (TLKM), and Unilever Indonesia (UNVR) (Table 1). Three main considerations guided the selection process. First, sectoral representation was ensured by covering industries such as finance, construction materials, agribusiness, telecommunications, and consumer goods, thereby allowing the optimization framework to be tested on heterogeneous risk–return profiles. Second, complete daily return data from January 2021 to December 2024, along with comprehensive sustainability reporting, facilitated the computation of consistent ESG scores. Third, the variability of ESG scores among the selected stocks, ranging from 0.20 to 0.80, provides a realistic and challenging testing environment for ESG-constrained portfolio optimization, where the algorithm must effectively balance financial performance objectives with sustainability requirements.

The ESG scores were derived from standardized KESGI indicators and subsequently normalized to a [0,1] scale, following the methodology outlined in Section 2.2. The variation in ESG scores reflects the heterogeneous sustainability profiles of the selected companies and introduces a significant degree of complexity into the portfolio optimization process, thereby reinforcing the necessity of employing an adaptive metaheuristic approach, namely the SOA, which can reconcile financial objectives with sustainability constraints. The mean return vector ($\bar{r}$) and the corresponding ESG score vector (s) were computed based on historical market data and publicly available sustainability reports, yielding:

Table 8. GWO results for ten stocks ESG portfolio optimization.

Stocks Code	GWO								
	(Repeated 1)			(Repeated 2)			(Repeated 3)		
	y_i	z_i	Risk	y_i	z_i	Risk	y_i	z_i	Risk
BBCA	0.0500	1	0.00095	0.0000	0	0.00083	0.0000	0	0.00083
BBRI	0.0520	1		0.2004	1		0.0760	1	
SMGR	0.0000	0		0.0000	0		0.0000	0	
DSNG	0.6862	1		0.5940	1		0.6174	1	
TLKM	0.0502	1		0.0000	0		0.0501	1	
UNVR	0.0000	0		0.0000	0		0.0524	1	
INDF	0.0500	1		0.0000	0		0.0512	1	
KLBF	0.0585	1		0.2056	1		0.0502	1	
TBIG	0.0530	1		0.0000	0		0.0504	1	
EXCL	0.0000	0		0.0000	0		0.0522	1	

Table 9. SOA for fifteen stocks ESG portfolio optimization.

Stocks Code	SOA		
	y_i	z_i	Risk
BBCA	0.0613	1	
BBRI	0.0504	1	
SMGR	0.0501	1	
DSNG	0.0711	1	
TLKM	0.0594	1	
UNVR	0.0502	1	
INDF	0.0596	1	
KLBF	0.0695	1	0.00033
TBIG	0.0539	1	
EXCL	0.0501	1	
PGAS	0.0613	1	
ANTM	0.0660	1	
WIKA	0.0500	1	
ICBP	0.1697	1	
BFIN	0.0776	1	

$$\bar{r} = \begin{bmatrix} -0.001627 \\ 0.006051 \\ 0.294360 \\ -0.021746 \\ 0.002127 \end{bmatrix}, \quad s = \begin{bmatrix} 0.2 \\ 0.4 \\ 0.6 \\ 0.8 \\ 0.5 \end{bmatrix}$$

The variance–covariance matrix Q, which captures both the individual volatilities of each asset and their mutual correlations, is expressed as:

$$Q = \begin{bmatrix} 0.000925 & 0.001337 & -0.027386 & 0.000063 & 0.000121 \\ 0.001337 & 0.004051 & -0.051406 & 0.000729 & 0.000180 \\ -0.027386 & -0.051406 & 2.042795 & -0.005887 & -0.006531 \\ 0.000063 & 0.000729 & -0.005887 & 0.003758 & -0.000086 \\ 0.000121 & 0.000180 & -0.006531 & -0.000086 & 0.000697 \end{bmatrix}$$

This integrated dataset, combining financial performance measures with sustainability indicators, serves as the empirical foundation for the ESG-constrained portfolio optimization model, ensuring that the analysis captures both return–risk dynamics and ESG considerations.

3.3. Ten-Stock Portfolio

Second, we use 10 stocks from the IDX ESG Leaders Index as the basis for portfolio construction. The stock selection process considers both the range of ESG scores, and the sectoral representation reported in the IDX ESG Index to ensure a diversified and representative sample (Table 2). By including companies across multiple sectors, the portfolio reflects the broader market composition while adhering to sustainability criteria. This selection approach ensures that the dataset captures varying levels of ESG performance, which is essential for evaluating the impact of ESG thresholds on portfolio optimization results. Using actual ESG scores allows the optimization model to more realistically reflect the trade-offs between financial performance and sustainability alignment, providing practical relevance for investors and policymakers.

We write the mean return of the 10 stocks in matrix $\bar{r}$ and ESG Score in matrix s, and variance–covariance matrix Q for ten stocks as follows

$$\bar{r} = \begin{bmatrix} 0.004920 \\ 0.007410 \\ 0.005230 \\ 0.008760 \\ 0.006810 \\ 0.005670 \\ 0.006430 \\ 0.007120 \\ 0.006850 \\ 0.005390 \end{bmatrix}, \quad s = \begin{bmatrix} 0.80 \\ 0.78 \\ 0.60 \\ 0.72 \\ 0.74 \\ 0.76 \\ 0.67 \\ 0.70 \\ 0.65 \\ 0.68 \end{bmatrix}$$

$$Q = \begin{bmatrix} 0.0012 & 0.0009 & 0.0007 & 0.0006 & 0.0008 & 0.0005 & 0.0004 & 0.0003 & 0.0002 & 0.0001 \\ 0.0009 & 0.0014 & 0.0008 & 0.0006 & 0.0007 & 0.0004 & 0.0003 & 0.0002 & 0.0003 & 0.0001 \\ 0.0007 & 0.0008 & 0.0011 & 0.0007 & 0.0006 & 0.0005 & 0.0004 & 0.0002 & 0.0002 & 0.0001 \\ 0.0006 & 0.0006 & 0.0007 & 0.0015 & 0.0007 & 0.0006 & 0.0003 & 0.0002 & 0.0003 & 0.0002 \\ 0.0008 & 0.0007 & 0.0006 & 0.0007 & 0.0016 & 0.0005 & 0.0004 & 0.0003 & 0.0004 & 0.0002 \\ 0.0005 & 0.0004 & 0.0005 & 0.0006 & 0.0005 & 0.0010 & 0.0003 & 0.0002 & 0.0002 & 0.0001 \\ 0.0004 & 0.0003 & 0.0004 & 0.0003 & 0.0004 & 0.0003 & 0.0012 & 0.0003 & 0.0002 & 0.0001 \\ 0.0003 & 0.0002 & 0.0002 & 0.0002 & 0.0003 & 0.0002 & 0.0003 & 0.0009 & 0.0001 & 0.0001 \\ 0.0002 & 0.0003 & 0.0002 & 0.0003 & 0.0004 & 0.0002 & 0.0002 & 0.0001 & 0.0008 & 0.0002 \\ 0.0001 & 0.0001 & 0.0001 & 0.0002 & 0.0002 & 0.0001 & 0.0001 & 0.0001 & 0.0002 & 0.0007 \end{bmatrix}$$

3.4. Fifteen-Stock Portfolio

Third, we also use a set of 15 constituent stocks from the IDX ESG Leaders Index as the empirical basis for the portfolio optimization problem. The selection of these assets was guided by two main criteria: (i) their inclusion in the IDX ESG Index, which ensures that they meet the minimum standards of ESG performance as assessed by the ISE and (ii) the variation in their ESG scores, which allows for the representation of different sustainability profiles across sectors. The 15 selected stocks cover a broad spectrum of industries, including banking, consumer goods, telecommunications, cement, agribusiness, infrastructure, and pharmaceuticals, thereby supporting the construction of a sectoral diversified portfolio that mitigates idiosyncratic risk.

We also use the ESG scores of these stocks to explicitly be incorporated into the portfolio optimization model as part of the constraint set. Specifically, a minimum portfolio-level ESG score is imposed to ensure that the selected allocation satisfies the sustainability requirement while simultaneously minimizing portfolio variance. This formulation transforms the optimization problem into a constrained mean–variance framework, where the objective function seeks to minimize equation (10) subject to budget, buy-in, and ESG compliance constraints. Incorporating ESG scores in this problem allows the resulting portfolio to achieve not only risk efficiency but also adherence to sustainable investment principles. We show the list of the 15 stocks, along with their corresponding ESG scores in Table 3.

We write the mean return of the 10 stocks in matrix $\bar{r}$ and ESG Score in matrix s, and variance–covariance matrix Q for ten stocks as follows:

$$\bar{r} = \begin{bmatrix} 0.0049 \\ 0.0074 \\ 0.0052 \\ 0.0088 \\ 0.0068 \\ 0.0057 \\ 0.0064 \\ 0.0071 \\ 0.0069 \\ 0.0054 \\ 0.0070 \\ 0.0065 \\ 0.0061 \\ 0.0072 \\ 0.0066 \end{bmatrix}, \quad s = \begin{bmatrix} 0.80 \\ 0.78 \\ 0.60 \\ 0.72 \\ 0.74 \\ 0.76 \\ 0.67 \\ 0.70 \\ 0.65 \\ 0.68 \\ 0.60 \\ 0.63 \\ 0.61 \\ 0.66 \\ 0.64 \end{bmatrix}$$

$$Q = \begin{bmatrix} 0.0013 & 0.0009 & 0.0008 & 0.0006 & 0.0009 & 0.0005 & 0.0006 & 0.0004 & 0.0003 & 0.0002 & 0.0003 & 0.0002 & 0.0003 & 0.0002 & 0.0002 \\ 0.0009 & 0.0016 & 0.0009 & 0.0007 & 0.0008 & 0.0004 & 0.0005 & 0.0003 & 0.0004 & 0.0002 & 0.0003 & 0.0003 & 0.0002 & 0.0002 & 0.0001 \\ 0.0008 & 0.0009 & 0.0015 & 0.0006 & 0.0007 & 0.0005 & 0.0004 & 0.0004 & 0.0002 & 0.0002 & 0.0002 & 0.0002 & 0.0002 & 0.0001 & 0.0001 \\ 0.0006 & 0.0007 & 0.0006 & 0.0017 & 0.0008 & 0.0006 & 0.0004 & 0.0002 & 0.0003 & 0.0002 & 0.0002 & 0.0003 & 0.0002 & 0.0002 & 0.0001 \\ 0.0009 & 0.0008 & 0.0007 & 0.0008 & 0.0018 & 0.0006 & 0.0005 & 0.0004 & 0.0003 & 0.0003 & 0.0003 & 0.0002 & 0.0003 & 0.0002 & 0.0002 \\ 0.0005 & 0.0004 & 0.0005 & 0.0006 & 0.0006 & 0.0011 & 0.0004 & 0.0003 & 0.0002 & 0.0002 & 0.0002 & 0.0002 & 0.0001 & 0.0002 & 0.0001 \\ 0.0006 & 0.0005 & 0.0004 & 0.0004 & 0.0005 & 0.0004 & 0.0013 & 0.0003 & 0.0002 & 0.0002 & 0.0003 & 0.0002 & 0.0002 & 0.0002 & 0.0001 \\ 0.0004 & 0.0003 & 0.0004 & 0.0002 & 0.0004 & 0.0003 & 0.0003 & 0.0010 & 0.0001 & 0.0002 & 0.0002 & 0.0001 & 0.0002 & 0.0001 & 0.0001 \\ 0.0003 & 0.0004 & 0.0002 & 0.0003 & 0.0003 & 0.0002 & 0.0002 & 0.0001 & 0.0009 & 0.0002 & 0.0002 & 0.0002 & 0.0001 & 0.0002 & 0.0001 \\ 0.0002 & 0.0002 & 0.0002 & 0.0002 & 0.0003 & 0.0002 & 0.0002 & 0.0002 & 0.0002 & 0.0008 & 0.0002 & 0.0002 & 0.0001 & 0.0002 & 0.0001 \\ 0.0003 & 0.0003 & 0.0002 & 0.0002 & 0.0003 & 0.0002 & 0.0003 & 0.0002 & 0.0002 & 0.0002 & 0.0010 & 0.0003 & 0.0002 & 0.0002 & 0.0001 \\ 0.0002 & 0.0003 & 0.0002 & 0.0003 & 0.0002 & 0.0002 & 0.0002 & 0.0001 & 0.0002 & 0.0002 & 0.0003 & 0.0012 & 0.0003 & 0.0002 & 0.0001 \\ 0.0003 & 0.0002 & 0.0002 & 0.0002 & 0.0003 & 0.0001 & 0.0002 & 0.0002 & 0.0001 & 0.0001 & 0.0002 & 0.0003 & 0.0011 & 0.0002 & 0.0001 \\ 0.0002 & 0.0002 & 0.0001 & 0.0002 & 0.0002 & 0.0002 & 0.0002 & 0.0001 & 0.0002 & 0.0002 & 0.0002 & 0.0002 & 0.0002 & 0.0009 & 0.0001 \\ 0.0002 & 0.0001 & 0.0001 & 0.0001 & 0.0002 & 0.0001 & 0.0001 & 0.0001 & 0.0001 & 0.0001 & 0.0001 & 0.0001 & 0.0001 & 0.0001 & 0.0008 \end{bmatrix}$$

All computational experiments in this research were conducted using MATLAB R2018a on an HP Pavilion 14-dv0067TX laptop equipped with an Intel Core™ i7 processor (4.70 GHz) and 8 GB RAM, operating under Windows 10. The experiments were designed to solve the MINLP formulation of the ESG-constrained portfolio optimization problem described in Section 2, incorporating both buy-in threshold rules and minimum ESG score requirements.

In this research, the minimum ESG score requirement was set to S_{min} is equals to 0.5 for minimum ESG requirement. This value reflects a moderate sustainability threshold, ensuring that the portfolio meets an acceptable level of ESG performance without being overly restrictive, thus allowing sufficient flexibility in asset selection. The target portfolio return was fixed at $R_p = 0.1952$, representing a realistic and attainable expected return level based on the historical performance of the selected assets, while maintaining compatibility with low-to-moderate risk profiles. The lower buy-in threshold l_i is equal to 0.05. It chooses to prevent negligible allocations that would be impractical in real-world investment settings, whereas the upper bound u_i equals to 1. It means to permits full allocation to a single asset if justified by its risk–return–ESG profile, thereby allowing maximum allocation flexibility.

To efficiently handle the MINLP formulation, a penalty function approach was adopted to transform the constrained problem into an unconstrained one. The penalty function $F(y,z)$ is given in Equation (10), which penalizes violations of the budget constraint, target return requirement, ESG score threshold, and buy-in threshold bounds. We used a penalty function approach, where very large numbers ($\rho = \mu = \gamma = 1000$) were assigned to discourage the algorithm from breaking fundamental rules such as budget, return, and ESG constraints. We use a smaller penalty value, $\alpha_i = \beta_i = 1000$, for buy-in threshold constraints because it can allow SOA to have some flexibility to test solutions near the boundaries without getting stuck too early. In simpler terms, this means the SOA was strongly guided to respect the main investment rules while still being free to explore different portfolio combinations. These values were determined based on preliminary experiments and existing literature [12][15], aiming to balance the magnitude of penalty terms relative to the original objective function. The large values of ρ, μ, γ ensure that violations of the fundamental constraints (budget, return, ESG) are strongly discouraged, effectively guiding the search process toward feasible solutions.

The penalty parameters were selected based on preliminary experiments and prior studies on constrained portfolio optimization. Large penalty values ($\rho = \mu = \gamma = 1000$) were assigned to budget, return, and ESG constraints to strictly enforce feasibility, while smaller penalties ($\alpha_i = \beta_i = 1000$) were used for buy-in thresholds to allow limited boundary exploration. A sensitivity test was conducted by varying penalty values within ±20%, and the resulting portfolio risks changed by less than 2%, indicating that the optimization results are stable with respect to penalty parameter selection.

The SOA parameters were selected based on prior empirical studies and fine tuning to the problem context, as presented in Table 4.

Initially, the SOA algorithm generated 100 random portfolio compositions that satisfied the specified lower and upper bound constraints. At each iteration, a new set of portfolios was produced by moving toward the current best solution following the spiral search pattern. We use penalty function for ESG portfolio optimization as in Equation (10) to find the ESG portfolio weight and risk. We also use GWO and PSO so that we can compare the results. Then, we show the results in Table 5. Based on the result in Table 5, we find that the SOA, GWO, and PSO give the same proportion investation and the minimum risk investation. The risk result equals to 0.8827 and we can choose the five asset to be invest. We can choose the DSNG with the proportion equals to 0.6630. After that, we can choose SMGR with the proportion equals to 0.1871. Then, we can continue choosing BBCA, TLKM, and UNVR because it have the same proportion value. It equals to 0.0500.

We continue using SOA, PSO and GWO for ten stocks ESG Portfolio Optimization with expected return portfolio equals to 0.0070. We show the results of three times repeated running SOA program in Table 6. Afterwards, we also do the same things into PSO and GWO. Then, we show the results in Tables 7 and 8. Based on the results in Table 6, we can see that SOA can give the consistent results into the same risk and the same proportion investment values for each repeated running program. The PSO and GWO results is random in repeated running program, not convergent into one value. We show it in Tables 7 and 8. Besides that, SOA results give the consistent results in repeated running program as show in Table 6 for three repeated running programs. Therefore, SOA result converges to the closest same results. We also find that, all stock in SOA results is chosen because the value of $z_i = 1$ for the three repeated running programs.

Based on the results in Table 7, we show PSO results for ten ESG portfolio problems. The PSO results giving the inconsistent risk value and not all stock is choosen in PSO results. In repeated 1, we can only choose the seven stock for invest in this portfolio. Next, in the repeated 2, we get only 1 stock not to be choosen in portfolio. Then, in the repeated 3, we get only 3 stocks not to be choosen in this portfolio. Based on Table 8, we describe PSO results for solving ten ESG portfolio problems. The PSO results giving the inconsistent risk value and not all stock is choosen in PSO results. In repeated 1, we can only choose the seven stock for invest in this portfolio. Next, in the repeated 2, we get 7 stocks not to be choosen in portfolio. Then, in the repeated 3, we get only 2 stocks not to be choosen in this portfolio.

Based on the results in Tables 6, 7, and 8, we find that SOA always converge to near closest value in three repeated running program and all stocks active or been choosen in this problems. Therefore, we use SOA for our next calculation in optimizing ESG-portfolio for fifteen stocks with target return portfolio equals to 0.0067. Thus, we show SOA results for solving ESG portfolio for fifteen stocks in Table 9. Based on the results in Table 9, we get the risk equal to 0.00033 with the fifteen stocks that can be chosen in this portfolio. We get ICBP having the highest proportion in this problem. It equals to 0.1697. Then, it follows by BFIN, DSNG, KLBF, BBCA and PGAS.

4. CONCLUSIONS

We solve mean-variance portfolio optimization problem with ESG and buy-in threshold constraints. We use real data from green economy stocks in Indonesia Stock Exchange. We formulate the ESG portfolio optimization problem into MINLP problems. We use SOA to solve this problem. Then, we validate and check effectivity of our SOA by comparing SOA results with PSO and GWO that has been used in this portfolio optimization. We do comparison for five and ten stocks. Based on the empirical results for five stocks, we show that SOA gives the same results with PSO and GWO results. Meanwhile for ten stocks, SOA gives the consistent results than the results of PSO and GWO. SOA successfully gives the consistent results and convergent into near closest value in the repeated running program results. Besides that, PSO and GWO give the inconsistent results because the results have change in every repeated running program, not converge to one value. Therefore, it can give an overview about SOA can be used in small number of stocks or extended stocks. Overall, the results demonstrate that SOA offers an effective computational approach to sustainable portfolio construction under practical investment constraints. The proposed framework maintains scalability and provides a robust foundation for future research, whereas we could expand this research by using hybrid optimization methods.

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Conflicts of Interest

The authors declare no conflict of interest.

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DECLARATION OF GENERATIVE AI

We hereby declare that the use of generative AI and AI-assisted technologies in the preparation of this manuscript is not applicable.

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