

# A Modified Dynamic Risk Multiplier for Managing Risky Assets Allocation in Unit-Linked Insurance Portfolios under the D-CPPI Strategy

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Received : May 23, 2025

Revised : September 15, 2025

Accepted : September 28, 2025

Online : November 3, 2025

## Abstract

Unit-linked insurance product (ULIPs) is a type of life insurance that provides two key benefits: protection (life) and investment. In this type of insurance, premiums are allocated to both risky and riskless assets, creating a portfolio that guarantees a minimum benefit. Therefore, it is essential to assess a portfolio's value based on its ability to meet the guaranteed minimum benefits, which can be done using the dynamic-constant proportion portfolio insurance (D-CPPI) strategy. The D-CPPI strategy employs a dynamic risk multiplier that adjusts according to the price movements of the risky asset. When the price of the risky asset increases, the multiplier also increases, and vice versa. If the proportion of risky assets in the portfolio is limited, a modified dynamic risk multiplier is needed. The objective of this strategy is to maximize a portfolio's value while minimizing risk. However, fluctuations in the price of risky assets may cause the portfolio value to fall below the guaranteed minimum, a risk known as gap risk. Hedging is required to mitigate the gap risk. This can be achieved by selecting and adjusting the initial dynamic risk multiplier in a suitable manner. The analysis presented in this paper uses actual data from the closing price of BBRI stocks from January 2017 to December 2021 as the risky asset. The results show that in a modified dynamic risk multiplier and hedging, the portfolio value consistently remains above or equal to the guaranteed minimum benefit.

**Keywords:** modified risk multiplier, dynamic cpqi, hedging strategies, portfolio valuation, risky asset allocation

## 1. INTRODUCTION

In the life insurance industry, there is a product that combines life insurance with investment, known as unit-linked insurance products (ULIPs). ULIPs offers the following dual benefits: protection (life coverage) and investment [1]. Unlike traditional life insurance, which pays out only upon a claim, ULIPs allow policyholders to benefit from the investment component regardless of whether a claim occurs, making them more appealing for those seeking both protection and wealth accumulation. Premiums collected from ULIPs are allocated between risky and riskless assets to form an insurance portfolio, denoted by  $V_t$ . The proportions of risky and riskless assets are continuously managed to maximize portfolio value while controlling risk. The attractiveness of a ULIP

increases when it guarantees a minimum benefit, or floor  $F_t$ , ensuring the policyholder receives at least a predetermined amount even during adverse market conditions. This poses a particular challenge for insurers, as the portfolio must be managed to ensure the ability to meet such guaranteed benefits [2]. Therefore, portfolio valuation is required to confirm that its value remains at least equal to the floor,  $V_t \geq F_t$ .

The portfolio of ULIPs consists of risky and riskless assets [3][4]. Its value is evaluated using various strategies, including constant proportion portfolio insurance (CPPI), time-invariant portfolio protection (TIPP), dynamic-CPPI (D-CPPI), and dynamic-TIPP (D-TIPP). CPPI is a foundational portfolio insurance strategy that was initially proposed by Perold in 1986 [5]. It gained popularity through the work of Black and Jones [6] for equity instruments and Perold and Sharpe [7] for asset allocation. CPPI has also been developed by Maalej and Prigent [8] using a stochastic dominance approach to analyze portfolio insurance in a low-interest-rate environment [9]. Furthermore, Khuman et al. extended the CPPI under cumulative prospect theory [10], while Hu et al. analyzed the mechanism of CPPI and optimized its performance [11]. CPPI has also been applied in value at risk based on portfolio insurance analysis in Alipour and Bastani [12].

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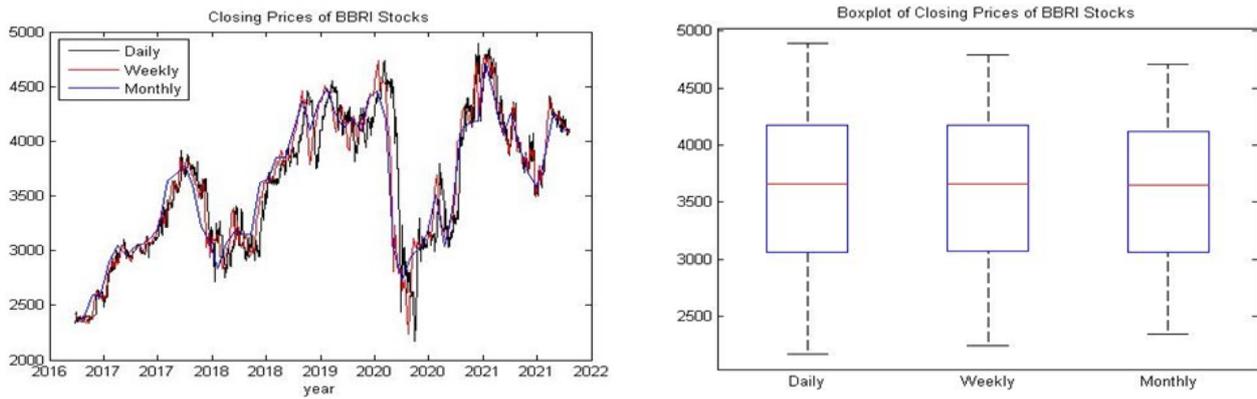
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**Figure 1.** Closing prices of BBRI stocks from 2017 to 2021 in daily, weekly, and monthly.

**Table 1.** Statistical summary of closing prices of BBRI from 2017–2021.

	Adjustment Periods		
	Daily	Weekly	Monthly
Mean	3,604.40	3,608.40	3,599.20
Median	3,660	3,660	3,650
Mode	3,050	3,030	3,850
Min	2,170	2,240	2,345
Quantile-1	3,060	3,070	3,062.50
Quantile-3	4,170	4,170	4,115
Max	4,890	4,790	4,710
Range	2,720	2,550	2,365
Standard deviation	631.9506	631.5644	607.8018
Skewness	-0.1299	-0.1474	-0.1795
Kurtosis	1.9867	1.9668	1.8897

TIPP was proposed in Estep and Kritzman publication [13] as a modification of the CPPI strategy [14]. The key difference lies in the assumption regarding the floor value,  $F_t$ . In the CPPI strategy, the floor grows use a riskless rate,  $F_t = e^{rt}F_0$ . In contrast, in TIPP, the floor is defined as the maximum of the previous floor and a constant proportion of the portfolio value at that time,  $F_t = \max(F_{t-1}, IV_t)$ . TIPP has been compared with CPPI in Kenneth and Eric paper [15]. The D-CPPI and D-TIPP strategies are extensions of CPPI and TIPP and incorporate a dynamic risk multiplier, as proposed by Yuan and Shanshan [16]. In this paper, we use the D-CPPI strategy to evaluate the portfolio value. The dynamic risk multiplier is adjusted according to the movement of the risky asset's price. Specifically, when the price of a risky asset increases, the dynamic risk multiplier also increases, and vice versa. This adjustment

maximizes the value of the portfolio while minimizing risk.

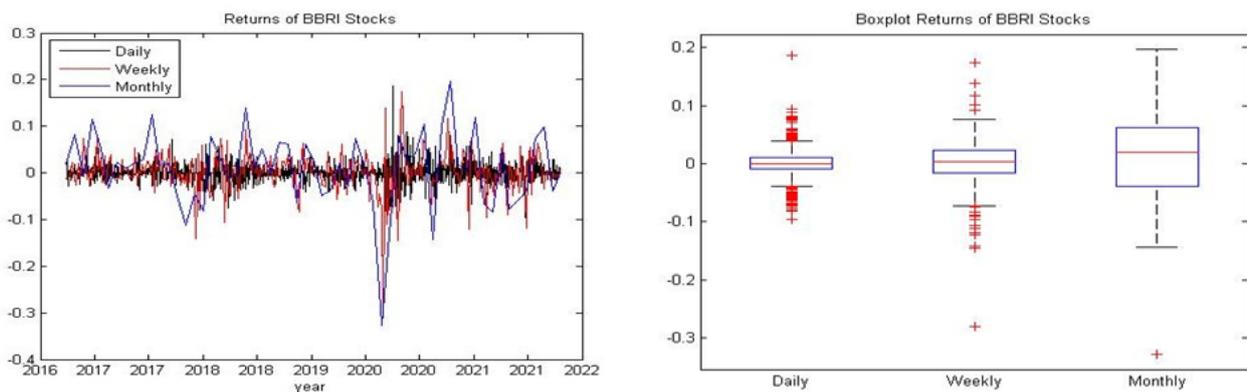
However, the portfolio value may fall below the floor or the guaranteed minimum benefit due to fluctuations in the price of the risky asset. This condition is referred to the gap risk, occurring when  $V_t < F_t$  [17]. Therefore, a hedging mechanism is necessary to mitigate this risk. For instance, Prigent and Tahar [18] extended CPPI by providing additional insurance for the cushion during a declining market, while Cont and Tankov [19] developed a jump-diffusion model to measure gap risk. Furthermore, Ameer and Prigent extended CPPI by introducing conditional floors to better control risk [20][21], and Xing et al. proposed a dynamic multiplier to mitigate gap risk [22]. Matenda developed a fuzzy differential equation with jumps to model gap risk [23], and Kalife and Mouti introduced hedging by purchasing put

options with a strike price of  $K_t = \left(1 - \frac{1}{m_t}\right) e^{rT} S_t$  while also selecting and adjusting the initial dynamic risk multiplier  $m_0 = \eta \frac{(\mu - i)}{\sigma^2}$  [24]. Fulli-Lemaire introduced a new class of dynamic trading strategies to address the hedging problem [25], and related hedging studies were also conducted in previous works [26]-[29].

Recent studies have further advanced the literature on CPPI, D-CPPI, and dynamic portfolio management, particularly in the context of ULIPs. Fitriawati et al. analyzing how frequent adjustments (daily, weekly, monthly) influence portfolio value [30]. El Farissi et al. applying a locally risk-minimizing hedging approach for ULIPs contracts [31]. Mancinelli and Oliva compared CPPI, TIPP, and exponential proportion PI, including fixed vs variable parameters, rebalancing frequency, and downside protection [32]. Guo and Melnikov

outlined option pricing with given risk constraints and its application to equity-linked life insurance contracts with guarantees [33]. Dupret and Hainaut explored portfolio insurance under rough volatility and Volterra processes, relevant for modeling real-market volatility that is heavy-tailed or exhibits memory effects [34]. Makkulau examines the role and effectiveness of combining insurance and hedging as complementary strategies for managing financing risk in volatile economic environments [35].

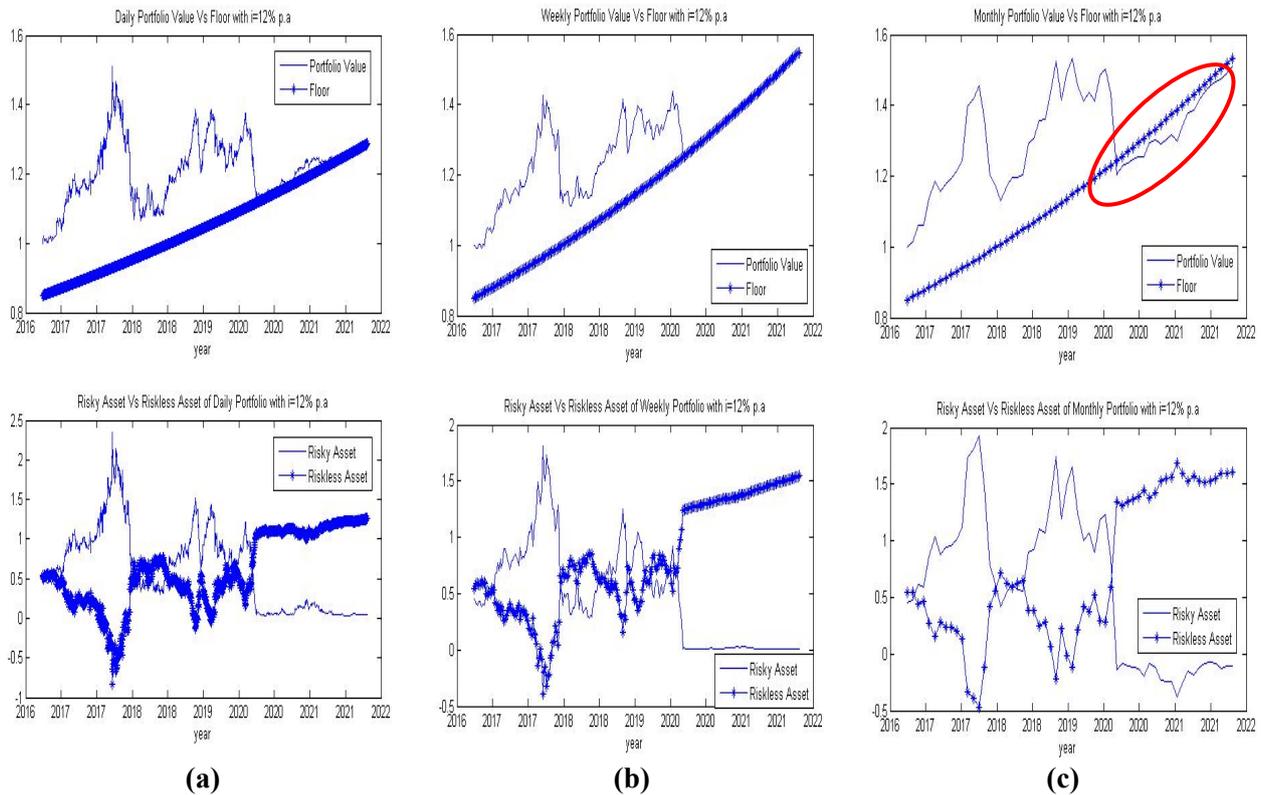
Although these studies have made significant contributions, several research gaps persist. First, most analyses are theoretical or simulation-based and lack real-world validation for ULIPs with guaranteed minimum benefits. Second, the impact of adjustment frequency on portfolio performance and risk mitigation requires further investigation.



**Figure 2.** Returns of BBRI stocks from 2017 to 2021 in daily, weekly, and monthly.

**Table 2.** Statistical summary of returns of BBRI stocks index from 2017 to 2021.

	Adjustment Periods		
	Daily	Weekly	Monthly
Mean	-0.000186	-0.000695	-0.0026
Median	0.000842	0.0016	0.0106
Mode	-0.0947	-0.2008	-0.1856
Min	-0.0947	-0.2008	-0.1856
Quantile-1	-0.0084	-0.0193	-0.0459
Quantile-3	0.0077	0.0207	0.0504
Max	0.1096	0.1136	0.0898
Range	0.2043	0.3144	0.2754
Standard deviation	0.019	0.037	0.066
Skewness	-0.1676	-0.8459	-0.7446
Kurtosis	8.7130	8.1521	3.0386



**Figure 3.** Portfolio value (above) and the proportion of risky and riskless assets (below) in daily (a), weekly (b), and monthly (c) basis under D-CPPI Strategy.

Third, inappropriate calibration of the dynamic risk multiplier can result in infeasible allocations, such as negative positions in risky or riskless assets, necessitating modifications to ensure practical applicability. In addition, the D-CPPI strategy dynamically adjusts asset allocation over time. This step begins by setting a floor equal to the guaranteed minimum benefit. Subsequently, a cushion is determined as the amount by which the portfolio value exceeds the floor. The amount allocated to the risky asset is equal to the dynamic risk multiplier multiplied by the cushion. The remainder of the portfolio value is then invested in the riskless asset. Under certain conditions, the value of risky or riskless assets can be negative. The value of riskless assets can become negative when the value of risky assets exceeds the portfolio value, which makes it impossible due to the available funds and the need to ensure that not all available funds are allocated to risky assets. Therefore, a modified dynamic risk multiplier is required to managing the proportion of risky assets. Therefore, this study aims to evaluate the portfolio performance under the D-CPPI strategy with a modified dynamic risk multiplier. Hedging is implemented if the portfolio value falls

below the floor by purchasing a put option and correctly choosing and adjusting the initial dynamic risk multiplier. In this context, allocations to risky assets are adjusted on a daily, weekly, or monthly basis. The results indicate that the portfolio value remains at least equal to the floor, thus ensuring that the portfolio can provide a guaranteed minimum benefit and avoid default.

## 2. MATERIALS AND METHODS

### 2.1. A Dynamic Risk Multiplier under the D-CPPI Strategy

The D-CPPI strategy is an extension of the CPPI strategy proposed by Yuan and Shanshan [16]. In the CPPI strategy, the risk multiplier is constant and is denoted as  $m$ . In contrast, the D-CPPI strategy employs a dynamic risk multiplier, denoted as  $m_t$ . Moreover, if the price of a risky asset increases, the dynamic risk multiplier also increases, and vice versa. Therefore, according to Yao and Li [36], the dynamic risk multiplier is given by Eq. (1).

$$m_t = m_{t-1} + a \ln \left( \frac{S_t}{S_{t-1}} \right), t = 1, 2, 3 \dots \quad (1)$$

Where  $m_t$  is a dynamic risk multiplier at time  $t$ ,  $S_t$  is the risky asset price at time  $t$ , and  $a$  is an amplifier determined by the investor’s preferred level of risk.

Eq. (1) shows that if the price of a risky asset increases, i.e.,  $S_t > S_{t-1}$ , then  $\ln\left(\frac{S_t}{S_{t-1}}\right)$  will be positive. As a result, the dynamic risk multiplier,  $m_t$ , increases proportionally to that of the amplifier  $a$ . Instead, when the price of a risky asset decreases, i.e.,  $S_t < S_{t-1}$ , the logarithmic term becomes negative, causing the dynamic risk multiplier  $m_t$  to decrease. This adjustment maximizes profits while minimizing risk.

Furthermore, the greater the value of  $a$ , the more sensitive the investor is to changes in the return and risk of the risky asset. According to Yao and Li [34], the portfolio value at time  $t$  is given by Eq. (2).

$$\begin{aligned} V_t &= E_t + R_t \\ C_t &= V_t - F_t \\ E_t &= m_t C_t \end{aligned} \tag{2}$$

Where  $V_t$  is portfolio value at time  $t$ ,  $E_t$  is the risky asset value at time  $t$ ,  $R_t$  the riskless asset value at time  $t$ ,  $C_t$  is the cushion value at time  $t$ , and  $F_t$  is the floor value at time  $t$ .

The initial floor is given by  $F_0 = lV_0$ , where  $l$  represents the proportion of the portfolio value that is guaranteed, as discussed in Matenda publication [20]. The floor then grows with the riskless interest rate, as described by equation  $F_t = e^{it}F_0$ , or

equivalently  $F_t = e^i F_{t-1}$ , where  $i$  is the riskless interest rate. It is important to note that the floor represents the guaranteed minimum benefit. The balancing of portfolio value continues until finished.

### 2.2. A Modified Dynamic Risk Multiplier under the D-CPPI Strategy

Based on Eq. (2), the value of either risky or riskless assets can be negative,  $\mathcal{E}t: E_t < 0$  or  $\mathcal{E}t: R_t < 0$ . The value of riskless assets can become negative,  $R_t < 0$ , when the value of risky assets exceeds the portfolio value,  $E_t > V_t$ , and vice versa. In fact, the value allocated to risky assets should not exceed the portfolio value,  $E_t \leq V_t$  in order to ensure sufficient funds, where  $V_t = E_t + R_t \geq 0$  at each time  $t$ . A large value of  $E_t$  is caused by a large risk multiplier,  $m_t$ . Therefore, the value of  $m_t$  Must be bound to ensure the condition is always met,  $E_t \leq V_t$  in Eq. (3).

$$\begin{aligned} E_t &\leq V_t \\ m_t C_t &\leq V_t \\ m_t &\leq \frac{V_t}{C_t} \end{aligned} \tag{3}$$

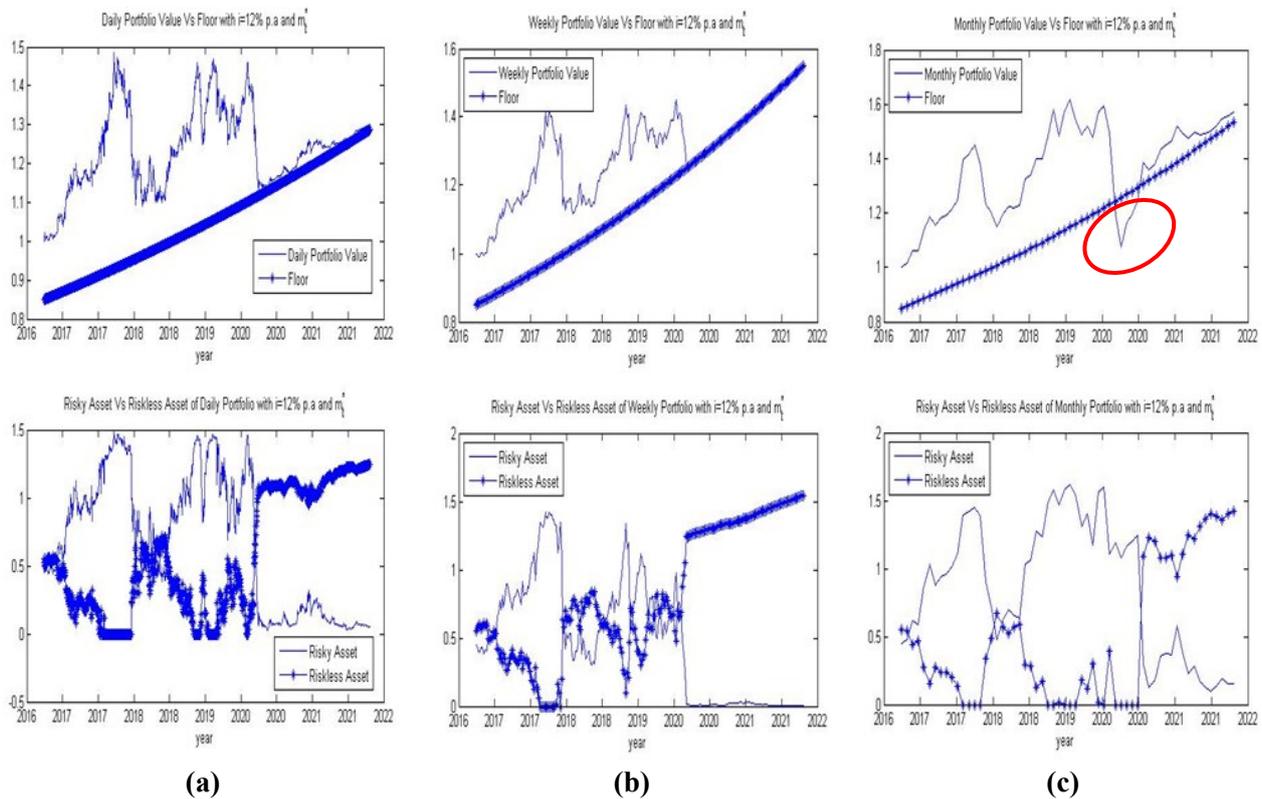
Thus, a modified risk multiplier is obtained in Eq. (4).

$$m_t^* = \min\left\{m_t, \frac{V_t}{C_t}\right\}, \quad t = 0, 1, 2, \dots \tag{4}$$

Based on Eq. (3), the value of risky assets can be equal to the portfolio,  $E_t = V_t$ . However, it is known

**Table 3.** Highlights the interplay among CPPI, D-CPPI, and modified D-CPPI.

	Allocation Rule	Feasibility ( $E_t, R_t \geq 0$ )	Downside Risk Control	Upside Participation	Practical Relevance for ULIPs
CPPI	$E_t = mC_t$ , $m = \text{constant}$	Always feasible if $m$ is bounded	Strong (conservative)	Limited (rigid multiplier)	Guarantees solvency but less competitive in bull markets
D-CPPI	$E_t = m_t C_t$ , $m_t = \text{dynamic}$	Not always (can produce $E_t > V_t$ or $R_t < 0$ )	Weaker (depends on volatility)	Strong (adaptive to market)	Attractive in theory, but prone to infeasible allocations
Modified D-CPPI	$E_t = m_t^* C_t$ or $E_t = m_t^{**} C_t$ , $m_t^*, m_t^{**} = \text{modified}$	Always feasible by design	Strong (bounded risk exposure)	Moderate–Strong (controlled flexibility)	Best trade-off: stable, feasible, and regulator-compliant



**Figure 4.** Portfolio value (above) and the proportion of risky and riskless assets (below) in daily (a), weekly (b), and monthly (c) basis under DCPPI strategy with  $m_t^*$ .

that the portfolio value should not be entirely allocated to risky assets. Therefore, the maximum proportion of risky assets must be restricted by  $\alpha$ , ensuring that the condition  $E_t \leq \alpha V_t$  remains valid. As a result, the risk multiplier in Eq. (4) must be modified to ensure that the condition is always satisfied in Eq. (5).

$$\begin{aligned}
 E_t &\leq \alpha V_t \\
 m_t C_t &\leq \alpha V_t \\
 m_t &\leq \frac{\alpha V_t}{C_t}
 \end{aligned}
 \tag{5}$$

Thus, a new modified risk multiplier is obtained in Eq. (6).

$$m_t^{**} = \min\left\{m_t, \frac{\alpha V_t}{C_t}\right\}, \quad t = 0,1,2,
 \tag{6}$$

If a gap risk condition occurs, that is,  $V_t < F_t$ , hedging is performed by selecting and adjusting the initial dynamic risk multiplier  $m_0 = \eta \frac{(\mu - i)}{\sigma^2}$ , as stated by Kalife and Mouti [22], where  $m_0$  is the initial dynamic risk multiplier,  $\eta$  is a scaling factor,  $\mu$  is the expected return of the risky asset,  $i$  is the riskless interest rate, and  $\sigma^2$  is the variance of the

return of the risky asset.

### 2.3. Data

The portfolio value was evaluated using the DCPPI strategy, with actual closing price data of BBRI stocks from January 2017 to December 2021 as risky asset. The evaluation considered different adjustment frequencies for the risky asset, specifically daily, weekly, and monthly rebalancing. The assumption was made that the premium is  $\wp = 1$ , paid once at the beginning of the investment period. Given the initial dynamic risk multiplier  $m_0 = 3$ , the interest rate  $i = 12\% p.a$ , the amplifier  $a = 2$ , and the floor proportion  $l = 85\%$ . In addition, assuming no acquisition fee, it leads to an initial portfolio value  $V_0 = 1$ . The data used in this simulation are the closing prices of BBRI stocks from January 2017 to December 2021, serving as the risky asset. The data are scheduled for analysis on a daily, weekly, or monthly basis, and are presented in Figure 1. Figure 1 shows the closing prices of the BBRI stocks from 2017 to 2021 on daily, weekly, and monthly were fluctuating. The highest price was recorded in 2021, while the lowest occurred in 2020. At the end of the

investment period, the prices were higher than at the beginning. The statistical summary of the data is presented in Table 1.

Figure 1 and Table 1 show that the data exhibit a slight left skew, characterized by a moderately longer tail on the left side. The mean and median values are close, while the mode is notably different, indicating the presence of outliers or a concentration of frequently occurring values at the lower or higher ends. The low kurtosis values suggests the data distribution has lighter tails than a normal distribution, with fewer extreme values or outliers. The data show substantial variability, as evidenced by the large range and standard deviation. Overall, the distribution is moderately spreads with slight negative skew and a relatively consistent central tendency.

Furthermore, the return of BBRI stocks was calculated using the formula in Highman (2004) work [37], as shown in Eq. (7).

$$r_{t+1} = \ln\left(\frac{S_{t+1}}{S_t}\right); t \geq 0 \tag{7}$$

where  $r_t$  is the return of the risky asset at time  $t$ , and  $S_t$  is the price of the risky asset at time  $t$ . Using the formula in Eq. (7), the return of the data is shown in Figure 2 and the statistical values of the returns are presented in Table 2.

Figure 2 and Table 2 show the data is slightly left-skewed, with smaller values occurring more frequently, although the mean is close to zero. There are outliers or extreme values, especially in the first two datasets, leading to high kurtosis

(heavy tails). The data shows a wide range of values, with more concentration in the first two datasets and a more spread-out distribution in the third dataset. In addition, the daily and weekly data are more centralized and consistent, while the monthly data exhibits more dispersion and is closer to a normal distribution. It can be stated that BBRI stock exhibits high volatility and frequently experiences extreme fluctuation, which may impact investment decisions depending on the investor's risk tolerance.

### 3. RESULTS AND DISCUSSIONS

#### 3.1. Portfolio Value under the D-CPPI Strategy

The portfolio values and the proportion of risky and riskless assets on the daily, weekly, and monthly basis under the D-CPPI strategy are shown in Figure 3. Figure 3 illustrates that negative values may arise for either the risky asset or the riskless asset. This occurs because the D-CPPI allocation rule, defined in Eq. (2). Specifically, a negative riskless asset position ( $R_t < 0$ ) emerges when  $E_t > V_t$ , i.e., when  $m_t C_t > V_t$ . In this case, the strategy attempts to allocate more than the total portfolio value to risky assets, which is not implementable in practice and corresponds to borrowing or shorting the riskless asset. Conversely, when  $C_t < 0$  (gap risk condition,  $V_t < F_t$ ), the rule produces  $E_t < 0$  for  $m_t > 0$ , implying a short position in the risky asset. Both outcomes reveal model inconsistencies when the multiplier becomes unbounded or the cushion turns negative.

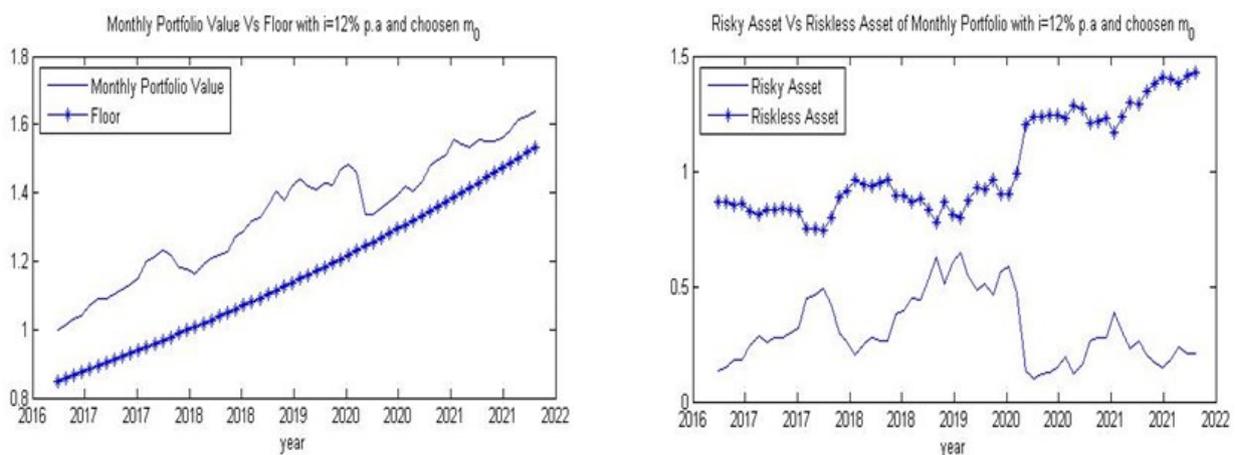
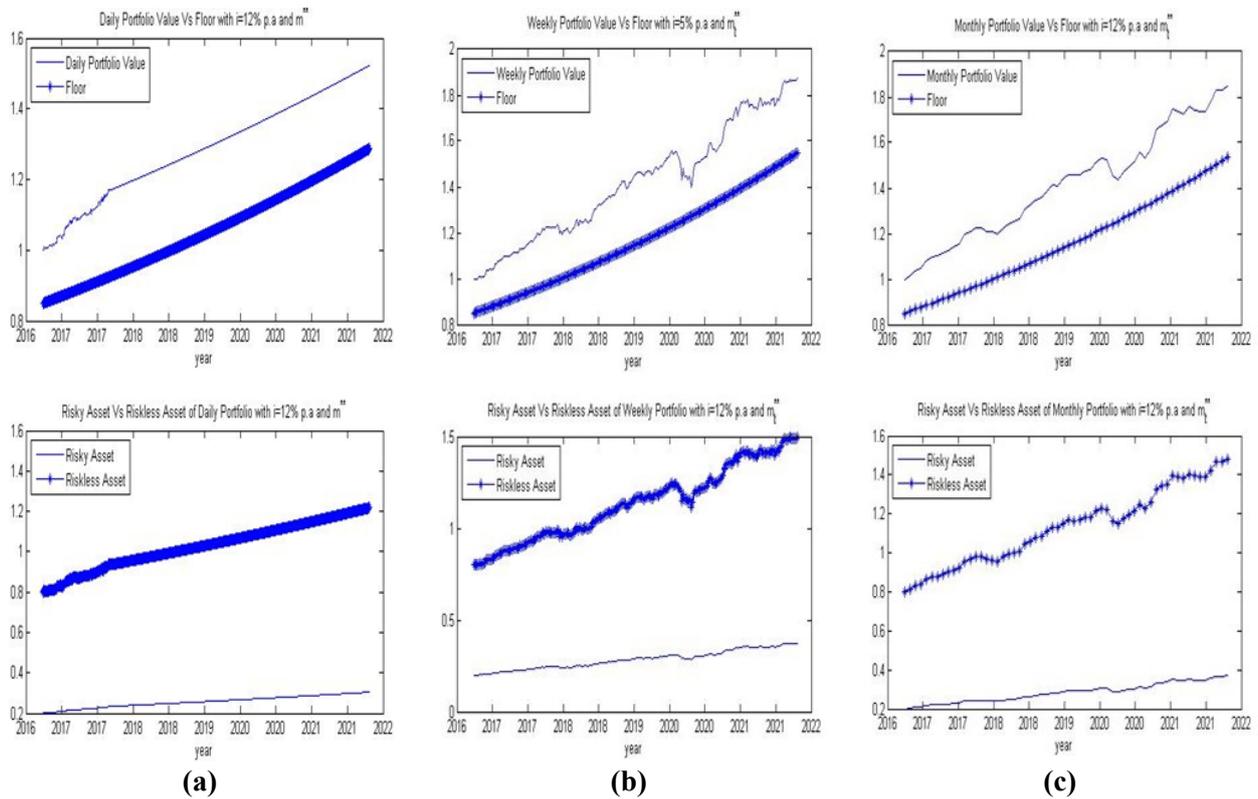


Figure 5. Portfolio value (left) and the proportion of risky and riskless assets (right) on monthly basis under DCPPI strategy with  $m_t^*$  and chosen  $m_0$ .



**Figure 6.** Portfolio value (above) and the proportion of risky and riskless assets (below) in daily (a), weekly (b), and monthly (c) basis under DCPPI strategy with  $m_t^{**}$ .

The strategic implication is that unconstrained D-CPPI may generate allocations that cannot be executed in practice and may exacerbate solvency risk for insurers offering ULIPs with guaranteed minimum benefits. Economically, while higher multipliers enhance upside participation in bullish markets, they also magnify downside exposure during market downturns, potentially driving the portfolio below the floor. The gap risk depicted within the circle in Figure 3(c), therefore, it is not merely a statistical anomaly but a structural vulnerability of the allocation rule. This explains why unconstrained D-CPPI, despite its flexibility, can lead to unrealistic trading behavior. By contrast, the classical CPPI strategy with a fixed multiplier avoids the issue of multiplier instability but lacks adaptability to changing market conditions, often resulting in lower portfolio performance during volatile periods. The D-CPPI improves responsiveness by adjusting the multiplier dynamically, which increases upside participation but also raises the probability of infeasible allocations and gap risk, as shown in Figure 3(c). To address these, a modified dynamic risk multiplier must be introduced to enforce feasibility.

This modification reduces the downside risk and prevents insolvency scenarios while still allowing dynamic adjustments to capture favorable market movements. For insurers managing ULIPs, this is crucial: the modified approach ensures that guaranteed minimum benefits can be met under adverse market conditions, while maintaining competitive performance in favorable markets. Furthermore, CPPI is robust yet conservative, D-CPPI is adaptive but prone to infeasibility, and the modified D-CPPI combines adaptability with practical solvency safeguards. This makes the modified version the most suitable for ULIPs portfolios, where regulatory compliance and protection of policyholder guarantees are as important as return optimization. Highlights the interplay among CPPI, D-CPPI, and modified D-CPPI in terms of feasibility, downside risk, and return participation are shown in Table 3. It shows that CPPI prioritizes solvency and policyholder guarantees but sacrifices competitiveness in bullish markets, whereas D-CPPI emphasizes performance but entails risks of insolvency and negative allocations. In contrast, the modified D-CPPI achieves a balance between the two, ensuring

feasibility (i.e., no negative assets) while maintaining adequate upside participation.

3.2. Portfolio Value under the D-CPPI Strategy with A Modified Dynamic Risk Multiplier  $m_t^*$

From Figure 3, the portfolio values and the proportion of risky and riskless assets on a daily, weekly, and monthly basis under the D-CPPI strategy were recalculated using  $m_t^*$ . The results are shown in Figure 4. Figure 4 illustrates that the values of both the risky asset and the riskless asset are no longer negative. This is because the value of the risky asset is controlled by  $m_t^*$  to ensuring it does not exceed the portfolio value. This ensures the adequacy of funds for allocation to both risky and riskless assets at each point in time. However, a gap risk condition still exists within the circle in Figure 4(c). This means that the portfolio value cannot meet its minimum guarantee (floor) at that time. Therefore, to address this gap risk condition, hedging is required by selecting and adjusting the initial dynamic risk multiplier  $m_0 = \eta \frac{(\mu - i)}{\sigma^2}$ . The results obtained from this hedging strategy are shown in Figure 5. Figure 5 illustrates that there is no longer any gap risk condition. Correctly choosing and adjusting the initial dynamic risk multiplier significantly reduces the downside risk. As a result, the portfolio value can meet the guaranteed minimum benefit at each point in time.

3.3. Portfolio Value under the D-CPPI Strategy with A Modified Dynamic Risk Multiplier  $m_t^{**}$

If the maximum percentage of the risky asset is limited to  $a = 20\%$  of the portfolio value, the portfolio values and the proportion of risky and riskless assets on a daily, weekly, and monthly basis under the D-CPPI strategy were recalculated from Figure 3 using  $m_t^{**}$ . The results are shown in Figure

6. Figure 6 illustrates that neither the risky asset nor the riskless asset exhibits negative values. Additionally, no gap risk condition is observed. This indicates that controlling and limiting the proportion of risky assets by  $m_t^{**}$  effectively mitigates the potential risks.

3.4. Portfolio vs. Floor

Based on the simulation results, the portfolio values are compared to the corresponding floor levels at the end of investment period. The portfolio values and floor levels at the end of investment period are presented in Table 4. The portfolio values at the end of the investment period under D-CPPI with  $m_t^{**}$  are higher than those under other strategies, whether daily, weekly, or monthly adjustment. Furthermore, the portfolio value at the end of the investment period in monthly adjustments is higher compared to the others, both under the D-CPPI and D-CPPI with  $m_t^*$ . In contrast, the portfolio value at the end of the investment period on weekly adjustments is higher than the others under the D-CPPI with  $m_t^*$ . As noted, the monthly adjustment under D-CPPI with  $m_t^*$  exhibits a gap risk conditions, as shown in Figure 4(c). When this gap risk is addressed by selecting and adjusting the initial dynamic risk multiplier  $m_0$ , the portfolio value at the end of the investment period reaches  $V_n = 1.63715$ . This suggests that monthly adjustments using the chosen  $m_0$  and  $m_t^*$  under the D-CPPI strategy result in a higher portfolio value at the end of the investment period compared to weekly adjustments. Thus, it can be concluded that controlling and limiting the proportion of risky assets using a modified dynamic risk multiplier significantly reduces downside risk and increases portfolio values.

Table 4. Portfolio values vs floor levels.

	Adjustment Periods					
	Daily		Weekly		Monthly	
	$V_n$	$F_n$	$V_n$	$F_n$	$V_n$	$F_n$
D-CPPI	1.29685	1.28667	1.55016	1.54880	1.50902	1.53339
D-CPPI with $m_t^*$	1.30005	1.28667	1.55025	1.54880	1.57048	1.53339
D-CPPI with $m_t^{**}$	<b>1.52200</b>	1.28667	<b>1.87320</b>	1.54880	<b>1.84859</b>	1.53339

## 4. CONCLUSIONS

In this study, the portfolio value is evaluated using the D-CPPI strategy, with particular emphasis on the impact of risky asset prices on portfolio performance. The adjustment frequency of risky asset allocations, whether on a daily, weekly, or monthly basis, is specifically examined. A modified dynamic risk multiplier is developed to control the proportion of risky assets in the portfolio. Modifying the dynamic risk multiplier in the D-CPPI strategy is essential because, in practice, an unconstrained multiplier may result in excessive allocations to risky assets that exceed the portfolio value, thereby causing the riskless asset to take on negative values and rendering the strategy unrealistic. Through this modification, the balance between risky and riskless assets can be maintained in line with the portfolio value, ensuring that the strategy remains feasible and applicable under real market conditions. Furthermore, in the event of gap risk, when the portfolio value falls below the floor level, hedging strategies are applied. Hedging against gap risk can be achieved through the careful selection and adjustment of the initial dynamic risk multiplier. This aspect is particularly crucial in the context of ULIPs, where insurers are obligated to provide a guaranteed minimum benefit to policyholders. The combination of a modified dynamic risk multiplier and hedging strategies has been shown to significantly reduce downside risk, thereby ensuring that the portfolio value remains sufficient to meet the guaranteed minimum benefit and avoid default. Insurers can adopt this modification to design ULIPs that are more stable and attractive. Ultimately, the modified multiplier serves as a key instrument for strengthening portfolio risk management amid dynamic and volatile market conditions.

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## Author Contributions

Conceptualization, S. W. I.; Methodology, Formal Analysis, and Investigation, A. F., S. W. I., K. N. S.; Validation and Supervision, S. W. I., and K. N. S.; Software, Resources, Data Curation, Writing – Original Draft Preparation, and Writing – Review & Editing, A. F.; Visualization, A. F., and K. N. S.

## Conflicts of Interest

The authors declare no conflict of interest.

## ACKNOWLEDGEMENT

This research is a part of the dissertation research of Andi Fitriawati and was supported by funds from the Indonesian Education Scholarship (BPI), Center for Higher Education Funding and Assessment (PPAPT) Kemendiktisaintek, and Indonesian Endowment Fund for Education (LPDP) awarded to Andi Fitriawati (grant number: 202101122065). Furthermore, Andi Fitriawati is also thankful to the ITB and Itera that supported her doctoral study at ITB.

## DECLARATION OF GENERATIVE AI

Not applicable.

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