

Optimization of Star Pomfret Feed Production as A Linear Programming Problem using A Hybrid Wolfe-Differential Evolution Algorithm

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Abstract

Star pomfret (*Trachinotus blochii*) is one of the most sought-after types of marine fish in Indonesia. The production of feed for star pomfret fish is an important factor because it is related to their survival and ability to grow well. Therefore, formulating the feed formulation for star pomfret (*Trachinotus blochii*) is very important to minimize feed production costs and ensure the nutritional adequacy of the fish. Therefore, we change the feed for star pomfret fish as a linear programming (LP) problem and solve it using the Hybrid Differential Evolution-Wolfe Algorithm (HWDEA). HWDEA combines the Wolfe method, which efficiently transforms constraints into a system of linear equations, with the use of the Differential Evolution Algorithm (DEA) to find a global optimization solution, which is a solution that is not trapped in a local minimum. We improve accuracy and efficiency by using HWDEA to find the optimal solution for this fish feed production. Our HWDEA can also overcome the limitations of traditional methods such as the simplex algorithm. Thus, we can show that HWDEA successfully reduced feed production costs from 12,353 IDR to 9,035 IDR per kg while maintaining nutritional balance. We can conclude that the HWDEA method successfully adapted to price fluctuations and raw material availability, allowing it to produce an optimal raw material composition in feed production. Therefore, HWDEA can be used as an efficient tool to provide significant cost savings for supporting sustainable and profitable fish farming.

Keywords: differential evolution algorithm, feed production, star pomfret, Wolfe algorithm

1. INTRODUCTION

Star pomfret fish (*Trachinotus blochii*) have several advantages over other sea fishes. First, its white and tender flesh makes pomfret fish is delicious and rich in protein. Besides that, pomfret fish also contains omega-3, therefore, this fish is good for our health [1] and has the opportunity for cultivation [2]. In terms of appearance, pomfret fish is attractive with its distinctive color and pattern, so it is often a choice in the appearance of dishes. Based on the price, this fish tends to be more expensive than some other seafood, but many people are willing to pay more because of its delicious taste [3]. Moreover, star pomfret is one of the marine fisheries commodities with high economic value and significant export demand [2].

According to previous work [4], four production

factors influence the success of cultivation, one of which is feed. Feed simultaneously influences freshwater fish production, which is the research object. The costs incurred for food production are one of the costs required for fish farming. These costs range from 60% to 70% of the total expenses incurred in cultivation activities because the fish need protein [5]. In another work [6], they stated that feed contains nutrients that impact metabolism, repair of damaged cells, and maintenance of the fish's body. A similar thing was also stated by the other researcher [5] about the quality of fish feed reviewed more deeply based on the nutritional composition contained therein. The fish need protein to grow fast, thus, marine fish can use artificial feed.

Lampung Marine Aquaculture Fisheries Center (BBPBL-Lampung) produced star pomfret feed independently. The production of star pomfret fish feed uses fish meal, soybean meal, wheat, meat and bone meal (MBM), bran, corn gluten meal (CGM), fish oil, and vitamin mix originating from within the country and abroad at low raw material prices. Different components serve as a reference for feed-cost components. Apart from that, pomfret feed must meet nutritional components consisting of a minimum of 36% protein, a minimum of 9% fat, a maximum of 13% ash, a maximum of 5% fiber, and a maximum of 10% water/moisture (BBPBL-

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Table 1. Ingredients for Star Pomfret Fish Feed Production in BBPBL-Lampung.

No	Ingredient	Composition (%)	Source of Ingredient	Local/ Import	Ingredient Price (IDR)	Feed Cost (IDR)
1	Fish meal	16	Lamongan, East Java	Local	20,000	3,200
2	Soybean meal	30	Tangerang, Banten	Import	10,000	2,600
3	Wheat flour	20	Cilegon, Banten	Local	8,000	1,600
4	Meat and bone meal (MBM)	10	South Lampung	Local	8,500	850
5	Rice bran	9	Boyolali, Central Java	Local	5,000	450
6	Corn gluten meal (CGM)	12	Cilegon, Banten	Local	12,000	1,440
7	Fish oil	2	Bogor, West Java	Local	20,000	400
8	Vitamin mix	1	Tangerang, Banten	Import	55,000	550
Total						11490
Packaging, Electricity, and Labor Costs/kg						863
Cost of Production						12.353

Lampung). Therefore, this research aims to minimize production costs so that greater profits from star pomfret feed production can be obtained through an optimization process completed using the hybrid Wolfe-differential evolution algorithm (HWDEA). As global demand for fishery products increases, optimization in fish feed production becomes important to reduce costs and maximize profit. In the star pomfret fish, as a fish with high economic value, the development of feed optimization techniques has experienced significant progress from conventional methods to computational and metaheuristic-based approaches. The differential evolution algorithm is a metaheuristic that is known as fast convergent and can find global optima as in Storn and Price [7] and Febrianti [8]. Then, Febrianti, et al [9][10] also used modification of differential evolution algorithm for solving some ordinary differential equations numerically and systems of ordinary differential equations. After that, Febrianti, et al [11] also use another modification of differential evolution algorithm for solving stiff ordinary differential equation. Furthermore, Febrianti, et al [11] also do modification of differential evolution algorithm for solving problems in finance such as approximate solution of Barrier option pricing [12] and vanilla option pricing [13]. Therefore, in this research, we also used hybrid Wolfe with differential algorithm to make the result of our research become more accurate. We use Wolfe method because it can efficiently transform constraints into a system of linear equations. Then, we use differential evolution algorithm for finding the global solution of this star pomfret feed production problems. This hybrid approach improves accuracy and efficiency, overcoming limitations of traditional methods such as the simplex algorithm.

At the initial stage, fish feed formulation, including star pomfret fish, is generally carried out using an experience-based approach (trial-and-error) and standard nutrition tables. This approach focuses on achieving a nutritional balance that meets the needs of fish at various stages of their life. However, this conventional method often does not consider cost factors optimally, so feed production costs can be very high. Conventional feed formulations are based on scientific knowledge about the nutritional needs of fish but pay less

attention to price variability and availability of raw materials. In addition, this method cannot quickly respond to changing market conditions or limited resources, so it often results in feed formulations that are expensive or less efficient in terms of production.

The development of optimization techniques for feed production began to experience a leap with the introduction of linear programming (LP). This LP method allows feed formulation to be carried out more systematically and efficiently by combining several available raw materials to achieve optimal nutritional composition at minimal cost. In the context of star pomfret fish feed, LP is used to determine the optimal proportion of ingredients such as fish meal, corn meal, oil, and premix based on the nutritional value and price of each ingredient. The LP assumes a linear relationship between the composition of raw materials and the nutrients produced. In this model, feed manufacturers can set nutritional limits that must be met (for example, a minimum of 30% protein and a maximum of 10% fat) and optimize raw material costs to obtain efficient feed formulations. However, although LP is effective for solving simple problems, this method has limitations in dealing with non-linear problems and increasing data complexity.

As complexity in feed production increases, particularly due to variability in raw materials, costs, and nutritional requirements, metaheuristic-based optimization techniques are starting to develop rapidly. Metaheuristics are search algorithms that can find optimal or near-optimal solutions to complex problems by exploring the solution space extensively. Some methods commonly used in fish feed formulation include the differential evolution algorithm, and multi-objective Bayesian optimization [14]. This method can handle a variety of non-linear constraints and works well in conditions, whereas traditional optimization methods, such as LP, experience limitations. These algorithms can be used for feed formulation problems to find better and more efficient solutions in a relatively short time.

One optimization technique that has shown great potential in fish feed formulation problems is the HWDEA. This algorithm uses a combination of two approaches: Wolfe's Algorithm, which is effective

in local exploitation, and differential evolution, which is powerful in global exploration of the solution space. HWDEA can solve optimization problems by minimizing feed raw material costs while ensuring that nutritional needs good feeding. This algorithm utilizes the exploratory approach of differential evolution, which allows for searching for optimal solutions in more ranges in solution space. Wolfe's algorithm works to increase efficiency in approaching optimal solutions through more intensive local searches. This hybrid approach is very suitable for pomfret fish feed formulation because this problem involves many non-linear variables and constraints, including varying raw material prices and minimum and maximum nutritional limits, like raw material availability constraints. HWDEA allows feed manufacturers to optimize the composition of raw materials by significantly minimizing costs compared to conventional optimization methods.

This research has novelty in calculating the production costs of star pomfret fish feed using a HWDEA. This research is unique in that it uses differential evolution as the final step of the Wolfe method to replace the simplex method, and no papers/journals/research discuss this. The differential evolution algorithm plays a role in optimizing linear programming problems built by the Wolfe method, replacing the simplex method, which is usually used in the final stage of the Wolfe method. The advantage of using differential evolution is that it can always be used to find optimal solutions to linear programming problems built using the Wolfe method in contrast to classical methods such as the simplex method. For this reason, hybridizing the Wolfe method with the differential evolution algorithm is necessary to obtain a more optimal solution. This differential evolution plays the role of finding the optimal solution to a system of linear equations built based on the Wolfe method.

2. MATERIALS AND METHODS

2.1. Formulation of Objective Function

The optimization problem can be defined as a LP problem with the following components: objective function is the cost of producing the feed, the minimizing of this function subject to the

nutritional requirements such as protein, fat, vitamins and minerals, ensuring that the feed meets the dietary needs of Star Pomfret (see Eq. 1–3).

$$\text{Minimize Cost} = \sum_{j=1}^n C_j x_j \tag{1}$$

$$\sum_{j=1}^n P_j x_j \geq P_{min} \quad (\text{protein requirement}) \tag{2}$$

$$\sum_{j=1}^n F_j x_j \geq F_{min} \quad (\text{Fat requirement}) \tag{3}$$

The C_j is the cost per unit of ingredient j , x_j is the quantity of ingredient j , Z is the total cost, P_j is the protein content of ingredient j , P_{min} is minimum of the protein requirement, F_j is the fat content of ingredient j , and F_{min} is minimum of the fat requirement. Similar formulations can be developed for vitamins, minerals, and other essential components. Other constraints include composition limits, cost, and non-negativity. For general formulation, definite Eq. (2) – (3) and other constraints as follows;

$$\sum_{j=1}^n a_{ij} x_j \geq b_i \tag{4}$$

$$x_j \geq 0; \tag{5}$$

where $i = 1, 2, \dots, m$ and $j = 1, 2, \dots, n$

2.2. Methods

2.2.1. Differential Evolution

This method is a very effective optimization algorithm, especially for problems in continuous, non-linear, and non-convex systems. It involves the evolution of a population of candidate solutions through mutation, crossover, and selection operators.

2.2.2. Wolfe Method

This method is a combination of the Kuhn-Tucker condition and simplex. It works by transforming inequality constraints into equations by inserting slack variables (s_i) for maximization problems and excess variables (e_i) for minimization problems. The linear programming problem (LPP) of the Kuhn-Tucker condition is then built for the same and an optimal solution for LPP is found, as is typically done using the simplex method. In this paper, the differential evolution algorithm will be used instead of the simplex method to enhance performance.

2.2.3. HWDEA

Combining the Wolfe line search with differential evolution can balance exploration and exploitation, overcoming some of the limitations of standard differential evolution. Steps of the hybrid algorithm are given as follows. At first, convert the inequality constraints Eq. (4) – (5) into equations by introducing the excess variables e_j in the i th constraints $i = 1, 2, \dots, n$. Then, derive the Karush-Kuhn-Tucker condition. Because this problem is a linear problem, we propose to remove the step of linearization in the Wolfe method. the Karush-Kuhn-Tucker conditions as follows:

$$\frac{\partial(\sum_{j=1}^n C_j x_j)}{\partial x_k} \delta_{jk} + \sum_{i=1}^m \lambda_i \left(\frac{\partial \sum_{j=1}^n a_{ij} x_j}{\partial x_k} \delta_{jk} - e_j \right) = 0 \tag{6}$$

$$\lambda_i \left(\sum_{j=1}^n a_{ij} x_j - b_i \right) = 0 \tag{7}$$

$$x_j \left[\frac{\partial(\sum_{j=1}^n C_j x_j)}{\partial x_k} \delta_{jk} + \sum_{i=1}^m \lambda_i \left(\frac{\partial \sum_{j=1}^n a_{ij} x_j}{\partial x_k} \delta_{jk} \right) \right] \tag{8}$$

where $x = (x_1, x_2, \dots, x_n)$, $e = (e_1, e_2, \dots, e_m)$, $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_m)$, δ_{jk} is delta function where $\delta_{jk} = 1$ for $k = j$, and $\delta_{jk} = 0$.

Third, use DE algorithm to obtain an optimum solution to LPP as follows:

Table 2. Nutritional Composition in Star Pomfret Fish Feed Production in BBPBL-Lampung.

No	Parameter	Star Pomfret Fish Feed Composition (%)
1	Protein	> 36
2	Fat	> 9
3	Ash	< 13
4	Fiber	< 5
5	Moisture	< 10

$$\text{minimize } Z = \sum_{j=1}^n C_j x_j \tag{9}$$

$$C_j + \sum_{i=1}^m \lambda_i a_{ij} - e_j = 0 \tag{10}$$

$$\lambda_i \left(\sum_{j=1}^n a_{ij} x_j - b_i \right) = 0 \tag{11}$$

$$x_j \left(C_j + \sum_{i=1}^m \lambda_i a_{ij} \right) = 0 \tag{12}$$

$$x_j, \lambda_i, e_j \geq 0 \tag{13}$$

where $i = 1, 2, \dots, m$ and $j = 1, 2, \dots, n$. Fourth, define the population size and initialize candidate solutions randomly within the constraints of the decision variables x_j . Afterward, update the initial population x_j through mutation, crossover, and selection processes.

Mutation: For each candidate solution x_j , DE generates a mutant vector v_j by combining other population members. The mutation process is expressed as:

$$v_j = x_{r1} + F \cdot (x_{r2} - x_{r2}) \tag{14}$$

where $r1, r2, r3$ are randomly chosen individuals from the population, and F is a scaling factor that controls the influence of the mutation.

Crossover: A crossover operation is performed between the current candidate solution x_j and the mutant vector v_j . The trial vector u_j is generated as follows:

$$u_j = \begin{cases} v_j; & \text{if } \text{rand}() < Cr \\ x_j; & \text{otherwise} \end{cases} \text{ and constraints are satisfied} \tag{15}$$

where Cr is the crossover probability, and $\text{rand}()$ is a random number between 0 and 1.

Selection: The trial vector u_j is evaluated. If it has a lower cost and satisfies the constraints, it replaces the current candidate solution x_j :

$$x_j = u_j; \text{ if } \min Z(u_j) < \min Z(x_j) \tag{16}$$

Finally, repeat the previous step for a predetermined number of iterations or until convergence criteria are met (to obtain optimal results).

2.3. Data Description

A model was developed using data from BBPBL-Lampung to create an objective function that can be used in the development of a feed production cost minimization model for star pomfret fish. The data on raw materials and nutritional requirements for producing star pomfret fish feed are summarized in [Tables 1 and 2](#).

[Table 2](#) will be referred to in developing constraints for the objective function, which consists of the eight ingredients detailed in [Table 1](#). The prices to be arrived at using this formulation may deviate from those presented in [Table 1](#) since the information was based on typical common references for the nutritive values of the eight ingredients in Indonesia. The nutritional content of the ingredients used at BBPBL-Lampung is classified as institutional confidential information and is not permitted for publication.

3. RESULTS AND DISCUSSIONS

3.1. Experiment Data

We re-construct the data in [Table 1](#) by incorporating the nutrient content requirements for star pomfret as specified in [Table 2](#). Four nutrient content limits—protein, fat, ash, and fiber—are used as shown in [Table 2](#). The water/moisture limit is excluded since it originates from sources outside the eight basic feed ingredients being considered. Furthermore, the four nutrient types are assumed to be derived from seven ingredients listed in [Table 1](#), excluding the vitamin Mix. The nutritional composition of these seven ingredients in star pomfret feed production is presented in [Table 3](#). The proportion of vitamin Mix in the feed mixture is assumed to be 1% based on the BBPBL-Lampung data in [Table 1](#).

We divided the optimization problem into two problems based on the protein, fat, ash, and fiber contents present in the ingredients listed in [Table 3](#). In problem 1, the percentages of protein, fat, ash, and fiber contents in the ingredients are determined by choosing the minimum value within the range provided in [Table 3](#). In problem 2, the percentages of protein, fat, ash, and fiber content in the ingredients are determined by calculating the maximum value within the given range. In problem 1, we can formulate a linear programming model

with the following minimum content requirements for seven ingredients listed in Table 3 to minimize the cost of producing star pomfret fish feed. Let data values '< 1' in the table be assumed to be 0.1. The objective is to minimize the total cost of the feed. Subject to this condition, nutritional requirements should be satisfied. Let x_1, x_2, \dots, x_7 represent the proportions (in percentage) of the following ingredients in the fish feed: x_1 corresponds to Fish Meal, x_2 to soybean meal, x_3 to wheat flour, x_4 to MBM, x_5 to rice bran, x_6 to CGM, and x_7 to fish oil. Minimize the total cost of the feed can be calculated by Eq. 17–22.

$$\text{Min Cost} = 200x_1 + 100x_2 + 80x_3 + 85x_4 + 50x_5 + 120x_6 + 200x_7 + 550 \tag{17}$$

Protein (min 36%)

$$0.60x_1 + 0.44x_2 + 0.10x_3 + 0.50x_4 + 0.12x_5 + 0.60x_6 + 0x_7 \geq 36 \tag{18}$$

Fat (min 9%)

$$0.05x_1 + 0.01x_2 + 0.01x_3 + 0.08x_4 + 0.13x_5 + 0.01x_6 + 0.99x_7 \geq 9 \tag{19}$$

Ash (max 13%)

$$0.10x_1 + 0.06x_2 + 0.005x_3 + 0.28x_4 + 0.08x_5 + 0.01x_6 + 0x_7 \leq 13 \tag{20}$$

Fiber (max 5%)

$$0.001x_1 + 0.03x_2 + 0.005x_3 + 0.001x_4 + 0.10x_5 + 0.01x_6 + 0x_7 \leq 5 \tag{21}$$

The total proportion of all ingredients (7 ingredient) must sum to 99%:

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 = 99 \tag{22}$$

Each proportion must be non-negative:

$$x_1, x_2, x_3, x_4, x_5, x_6, x_7 \geq 0 \tag{23}$$

In problem 2, we reformulate a linear programming model to minimize the cost of producing star pomfret fish feed making use of the maximum content requirements for seven ingredients in Table 3. Minimize the total cost of the feed can be calculated by Eq. 24–28.

$$\text{Min Cost} = 200x_1 + 100x_2 + 80x_3 + 85x_4 + 50x_5 + 120x_6 + 200x_7 + 550 \tag{24}$$

Table 3. Nutritional Composition of Ingredients for Star Pomfret Fish Feed Production.

No	Ingredient	Variable Composition	Protein (%)	Fat (%)	Ash (%)	Fiber (%)	Ingredient Price (IDR)
1	Fish Meal	x_1	60–72	5–12	10–20	< 1	20,000
2	Soybean Meal	x_2	44–48	1–2	6–7	3–7	10,000
3	Wheat Flour	x_3	10–12	1–2	0.5–1.0	0.5–1.0	8,000
4	MBM	x_4	50–55	8–12	28–36	< 1	8,500
5	Rice Bran	x_5	12–14	13–15	8–10	10–13	5,000
6	CGM	x_6	60–65	1–3	1–2	1–3	12,000
7	Fish Oil	x_7	-	99	-	-	20,000
8	Vitamin Mix	1%	-	-	-	-	55,000
Total		100					

Protein:
 $0.72x_1 + 0.48x_2 + 0.12x_3 + 0.55x_4 + 0.14x_5 + 0.65x_6 + 0x_7 \geq 36$ (25)

Fat:
 $0.12x_1 + 0.02x_2 + 0.02x_3 + 0.12x_4 + 0.15x_5 + 0.03x_6 + 0.99x_7 \geq 9$ (26)

Ash:
 $0.20x_1 + 0.07x_2 + 0.01x_3 + 0.36x_4 + 0.10x_5 + 0.02x_6 + 0x_7 \leq 13$ (27)

Fiber:
 $0.01x_1 + 0.07x_2 + 0.01x_3 + 0.01x_4 + 0.13x_5 + 0.03x_6 + 0x_7 \leq 5$ (28)

The proportions of each of the seven ingredients, derived in that a total proportion of all the ingredients satisfies Eq. (22), must be non-negative, as in Eq. (23) above. This model can be solved using linear programming techniques or optimization solvers, such as the HWDEA, to determine the optimal ingredient proportions that minimize feed production costs while satisfying all nutritional requirements.

3.2. Wolfe Method and Differential Evolution

For the problems 1 and 2, the HWDEA is used for finding an optimal solution. The Wolfe method gives the first step in the process whereby a linear problem is provided with the constraints which are presented as a system of linear equations. The differential evolution algorithm follows after to determine the solution.

3.2.1. Wolfe Method

This method is a combination of the Karush-Kuhn-Tucker condition and the Simplex method. This research only adopted the Wolfe procedure to convert inequality constraints into a system of linear equations. The excess e_j variables with $j = 1, 2, \dots, 7$ are introduced to convert the inequality constraint into equality. These excess variables are added To transform "less than or equal to" (\leq) constraints into equalities, and they are subtracted from "greater than or equal to" (\geq) constraints.

For problem 1, the minimization problem with Karush-Kuhn-Tucker conditions can be written as:

$$\text{minimize } Z = 200x_1 + 100x_2 + 80x_3 + 85x_4 + 50x_5 + 120x_6 + 200x_7 + 550 \quad (29)$$

$$200 - 0.60\lambda_1 - 0.05\lambda_2 + 0.10\lambda_3 + 0.001\lambda_4 - \lambda_5 - e_1 = 0 \quad (30)$$

$$100 - 0.44\lambda_1 - 0.01\lambda_2 + 0.06\lambda_3 + 0.03\lambda_4 - \lambda_5 - e_2 = 0 \quad (31)$$

$$80 - 0.10\lambda_1 - 0.01\lambda_2 + 0.005\lambda_3 + 0.005\lambda_4 - \lambda_5 - e_3 = 0 \quad (32)$$

$$85 - 0.50\lambda_1 - 0.08\lambda_2 + 0.28\lambda_3 + 0.001\lambda_4 - \lambda_5 - e_4 = 0 \quad (33)$$

$$50 - 0.12\lambda_1 - 0.13\lambda_2 + 0.08\lambda_3 + 0.10\lambda_4 - \lambda_5 - e_5 = 0 \quad (34)$$

$$120 - 0.60\lambda_1 - 0.01\lambda_2 + 0.01\lambda_3 + 0.01\lambda_4 - \lambda_5 - e_6 = 0 \quad (35)$$

$$200 - 0.99\lambda_2 - \lambda_5 - e_7 = 0 \quad (36)$$

$$\lambda_1(36 - 0.60x_1 - 0.44x_2 - 0.10x_3 - 0.50x_4 - 0.12x_5 - 0.60x_6) = 0 \quad (37)$$

$$\lambda_2(9 - 0.05x_1 - 0.01x_2 - 0.01x_3 - 0.08x_4 - 0.13x_5 - 0.01x_6 - 0.99x_7) = 0 \quad (38)$$

$$\lambda_3(0.10x_1 + 0.06x_2 + 0.005x_3 + 0.28x_4 + 0.08x_5 + 0.01x_6 - 13) = 0 \quad (39)$$

$$\lambda_4(0.001x_1 + 0.03x_2 + 0.005x_3 + 0.001x_4 + 0.10x_5 + 0.01x_6 - 5) = 0 \quad (40)$$

$$\lambda_5(99 - x_1 - x_2 - x_3 - x_4 - x_5 - x_6 - x_7) = 0 \quad (41)$$

$$x_1(200 - 0.60\lambda_1 - 0.05\lambda_2 + 0.10\lambda_3 + 0.001\lambda_4 - \lambda_5) = 0 \quad (42)$$

$$x_2(100 - 0.44\lambda_1 - 0.01\lambda_2 + 0.06\lambda_3 + 0.03\lambda_4 - \lambda_5) = 0 \quad (43)$$

$$x_3(80 - 0.10\lambda_1 - 0.01\lambda_2 + 0.005\lambda_3 + 0.005\lambda_4 - \lambda_5) = 0 \quad (44)$$

$$x_4(85 - 0.50\lambda_1 - 0.08\lambda_2 + 0.28\lambda_3 + 0.001\lambda_4 - \lambda_5) = 0 \quad (45)$$

$$x_5(50 - 0.12\lambda_1 - 0.13\lambda_2 + 0.08\lambda_3 + 0.10\lambda_4 - \lambda_5) = 0 \quad (46)$$

$$x_6(120 - 0.60\lambda_1 - 0.01\lambda_2 + 0.01\lambda_3 + 0.01\lambda_4 - \lambda_5) = 0 \quad (47)$$

$$x_7(200 - 0.99\lambda_2 - \lambda_5) = 0 \quad (48)$$

$$x_1, x_2, x_3, x_4, x_5, x_6, x_7 \geq 0 \quad (49)$$

$$\lambda_1, \lambda_2, \dots, \lambda_5 \geq 0 \quad (50)$$

$$e_1, e_2, e_3, e_4, e_5, e_6, e_7 \geq 0 \quad (51)$$

where $\lambda_1, \lambda_2, \dots, \lambda_4$ correspond to the nutritional constraints, and λ_5 correspond to total proportion constraint of all ingredients (7 ingredients).

The constraints (30)–(36) and (42)–(48) give the following term.

$$x_1e_1 = 0, x_2e_2 = 0, x_3e_3 = 0, x_4e_4 = 0, x_5e_5 = 0, x_6e_6 = 0, x_7e_7 = 0 \quad (52)$$

Since to are assumed to be non-zero, then;

$$e_1 = e_2 = e_3 = e_4 = e_5 = e_6 = e_7 = 0 \quad (53) \quad \text{and}$$

so, Equation (29) is minimized with constraints: $\text{Cost} = 9660 \quad (74)$

$$200 - 0.60\lambda_1 - 0.05\lambda_2 + 0.10\lambda_3 + 0.001\lambda_4 - \lambda_5 = 0 \quad (54)$$

$$100 - 0.44\lambda_1 - 0.01\lambda_2 + 0.06\lambda_3 + 0.03\lambda_4 - \lambda_5 = 0 \quad (55)$$

$$80 - 0.10\lambda_1 - 0.01\lambda_2 + 0.005\lambda_3 + 0.005\lambda_4 - \lambda_5 = 0 \quad (56)$$

$$85 - 0.50\lambda_1 - 0.08\lambda_2 + 0.28\lambda_3 + 0.001\lambda_4 - \lambda_5 = 0 \quad (57)$$

$$50 - 0.12\lambda_1 - 0.13\lambda_2 + 0.08\lambda_3 + 0.10\lambda_4 - \lambda_5 = 0 \quad (58)$$

$$120 - 0.60\lambda_1 - 0.01\lambda_2 + 0.01\lambda_3 + 0.01\lambda_4 - \lambda_5 = 0 \quad (59)$$

$$200 - 0.99\lambda_2 - \lambda_5 = 0 \quad (60)$$

In problem 2, the same procedure is applied resulting in a minimization problem, which is Equation (29) subject to the following constraints:

$$200 - 0.72\lambda_1 - 0.12\lambda_2 + 0.20\lambda_3 + 0.01\lambda_4 - \lambda_5 = 0 \quad (61)$$

$$100 - 0.48\lambda_1 - 0.02\lambda_2 + 0.07\lambda_3 + 0.07\lambda_4 - \lambda_5 = 0 \quad (62)$$

$$80 - 0.12\lambda_1 - 0.02\lambda_2 + 0.01\lambda_3 + 0.01\lambda_4 - \lambda_5 = 0 \quad (63)$$

$$85 - 0.55\lambda_1 - 0.12\lambda_2 + 0.36\lambda_3 + 0.01\lambda_4 - \lambda_5 = 0 \quad (64)$$

$$50 - 0.14\lambda_1 - 0.15\lambda_2 + 0.10\lambda_3 + 0.13\lambda_4 - \lambda_5 = 0 \quad (65)$$

$$120 - 0.65\lambda_1 - 0.03\lambda_2 + 0.02\lambda_3 + 0.03\lambda_4 - \lambda_5 = 0 \quad (66)$$

$$200 - 0.99\lambda_2 - \lambda_5 = 0 \quad (60)$$

$$\lambda_1(36 - 0.72x_1 - 0.48x_2 - 0.12x_3 - 0.55x_4 - 0.14x_5 - 0.65x_6) = 0 \quad (67)$$

$$\lambda_2(9 - 0.12x_1 - 0.02x_2 - 0.02x_3 - 0.12x_4 - 0.15x_5 - 0.03x_6 - 0.99x_7) = 0 \quad (68)$$

$$\lambda_3(0.20x_1 + 0.07x_2 + 0.01x_3 + 0.36x_4 + 0.10x_5 + 0.02x_6 - 13) = 0 \quad (69)$$

$$\lambda_4(0.01x_1 + 0.07x_2 + 0.01x_3 + 0.01x_4 + 0.13x_5 + 0.03x_6 - 5) = 0 \quad (70)$$

3.2.2. Differential Evolution

The differential evolution algorithm will be used to solve problems 1 and 2 with the following parameters: Max_Iter = 2000, NP = 15, F = 0.5, and Cr = 0.9. For problem 1, we have the results of problem 1 using differential evolution are

$$x_1 = 10, x_2 = 4, x_3 = 6, x_4 = 29, x_5 = 42, x_6 = 7, x_7 = 1 \quad (71)$$

and

$$\text{Cost} = 9035 \quad (72)$$

For problem 2, we have the results using differential evolution of

$$x_1 = 8, x_2 = 19, x_3 = 13, x_4 = 28, x_5 = 23, x_6 = 7, x_7 = 1 \quad (73)$$

In this research, we use MATLAB R2018a in HP Pavilion Laptop Model 14-dv0067TX that is equipped with processor Intel Core TM i7- 4.70 GHz with 8 GB ram. We build a linear program with constraints in the form of a system of equations by applying the Wolfe method. Then, we solve the problem using the differential evolution algorithm. In this problem, the system of equations of constraints cannot be solved using the Simplex method, so the HWDEA approach is essential for addressing this problem. The system of equations in problems 1 and 2 can have many possible solutions, but applying differential evolution to the running process 20 times with an average running time of less than 3 seconds obtains optimal results, as in Equations (71)–(74).

The different proportions of results, in Problems 1 and 2 with the BBPBL-Lampung data, are possibly due to differences in some of the nutrient composition of the ingredients listed in Table 3 that were used by BBPBL-Lampung. However, the results showed that the cost of the formulated feed is not much different from the price of BBPBL-Lampung, which indicates that the nutrient of the utilized ingredients by BBPBL-Lampung is not significantly different from the ingredients used in Table 3. The cost of the formulated feed in our results is slightly lower than the price at BBPBL-Lampung. This suggests that BBPBL-Lampung has the potential to reformulate ingredients with an optimized nutrient composition that can result in a more economical formulated feed.

4. CONCLUSIONS

We have shown detailed steps of a HWDEA to optimize the star fish feed production by changing the problem into a linear programming problem with constraints defined by a system of linear equations. We can extend this model to other fish feed productions. Thus, we can get a minimum cost in fish feed production. The reason for using the hybrid method in finding this optimal solution is that the results with the classic method (Wolfe) may not be optimal enough or cannot be used to find an

optimal solution to the problem. Therefore, we use the hybrid method called a HWDEA. The results show that HWDEA can find the optimal solution for the problem. Then, the future works of this research are developing implications of the fish food model by considering fish growth, which was completed by exploring the HWDEA.

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Conflicts of Interest

The authors declare no conflict of interest.

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