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Research Article

A Mathematical Model of an Electoral Process and Predicting of Outcome

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Abstract

We developed and analysed a mathematical model to study the dynamics of an electoral process and predict the outcome in Nigeria with three political parties as major actors. We model the ruling party (P_1); major opposition party (P_2) and minority opposition parties (P_3). The model includes party campaigners of the three political parties and a class of eligible voters. The model also incorporates the Independent National Electoral Commission (INEC), electoral observers, judiciary, security personnel and thugs. The influx of voters into the system is determined by the registration rate of voter cards. The model's interest lies in the measure of positive influence that the party campaigners woo eligible voters to join their party. With this, the model mirrors election's outcome and the movement of voters between the three groups of political parties involved. The model has proven that the pattern of election's outcome for each political party is affected by the defection of party members and lack of policy consistency. We determine the equilibria analytically and discuss the stability of the system. Numerical simulations are graphed in combination with curve fitting to compare each party's performances over the years. The result shows that no political party remains in power steadily from 2007 to 2019. Thus, the prediction of elections outcome to the political space of any region in Nigeria depends on political party structures and voters' decisions.

Keywords: election, prediction, mathematical model, eligible voters, political parties, campaigners, INEC, judiciary

1. INTRODUCTION

Globally, political elections are more of a chameleon of colours with uncertainties and conflicting interests. A democratic system of government provides a vehicle by which the people and citizens directly retain and exercise political independence [1]. Democracy is thus unique in the fact that the power of government finds its source in the people. The fulcrum point in democracies is competitive elections. Elections are decision-making processes which enable a population to choose an individual to occupy a formal office. In a democracy, offices in the executive, and parliament legislative arms of government are filled by engaging this mechanism [2].

The study aims to develop and analyse a mathematical model for predicting election's outcome amongst three groups of political parties in Nigeria. The specific objectives are to obtain the equilibrium states of the model equations; carry out stability analysis of the equilibrium states of the model; obtain the analytical solution of the model equations; determine the bounds of the electoral model; carry out sensitivity analysis on the model parameters and numerical simulations with curve fittings to present the results graphically; and to discuss the research findings in a simple language.

In line with this, we utilised permutations of mathematical applications to study the dynamics of electoral outcomes over the years in Nigeria through modelling and data forecasting. Modelling real life situations using mathematical equations starts with close examination, investigation, and projection. Election's outcome is one of the real-life problems that is highly sensitive and nonlinear. Therefore, a mathematical model depicting this problem is also nonlinear. The voting system is a single vote method otherwise the voter would be disenfranchised and there are three major groups of political parties. All the political parties contact the voter class to join their party. However, the unwholesome defection of individuals from one political party to the other has become a common trend for many reasons. This is a kind of Monte-

Carlo game where some of the party members who do not get proper positions or weight in their party or are not very much satisfied with the changes occurring in the party's ideology, leave the party and join another one in order to achieve their desire. Thus, election's outcome is a sensitive process that needs a scientific approach. This research work is essentially a theoretical experiment that provides answers to probing questions on election's outcome.

Governance and political leadership are a crucial aspect of human existence. In a democratic country like Nigeria, very often, political officeholders and elected parliaments make decisions that are constitutionally binding on all citizens irrespective of ethnicity, religious affiliation or skin colour. In Nigeria, INEC is saddled with the responsibility of conducting elections to make a pronouncement of individuals who win elective positions. The process pivoted eligible voters as important decision-makers. Whether or not it is possible to assess the policy positions of voters and candidates, there are hidden variables that affect a voter's choice in common cases. Therefore, policy positions are always measured with error, especially in Nigeria democratic setting where political parties lack policy and ideology. A voter's behaviour depends on either policy component, candidate integrity or imperfect information. Voters normally consider how the policy platform affects their utility such as welfare, cost of living and foreign policy and also, have sympathy towards a candidate for scandals or feelings. Candidates do not know with certainty the voters sympathy. Voters do not show preferences based on the policy implemented by the winner but also on the candidate's personality and identity [3]. The average popularity of candidates before the election also affects voter's decisions. Popularity before elections is out of candidates' control as the outbreak of scandals or other news may alter the popularity of a candidate, to the advantage of their opponents.

There are many reasons for voters' uncertainty about the candidates [4]. A candidate policy positions may be imperfectly perceived or may be perceived as a random variable. Uncertainty about new developments and future events may also complicate voter's decision. Hinich et al [5], they study a theory of electoral equilibrium whereby, future-oriented voters accept unavoidable uncertainty, even if they are confident about the candidate's current views on policy issues. On the flip side, a candidate is faced with the dilemma of never grasping the full range of factors that affect citizens' vote decisions. Having rational, informed, voters who have well-articulated views on policy issues, is not sufficient for the candidate to ascertain how the votes will be cast. In addition, the data he possesses are likely to contain a large amount of error.

Some modelling studies have been conducted regarding the growth of political parties and voters. In most of the studies, statistical methodology is used by considering that two political parties are competing for the voters' class [6]-[9]. In particular, All Progressives Congress (APC) and People Democratic Party (PDP) in Nigeria compete for the votes of undecided voters [10][11]. They have considered the role of election campaigns by all competing candidates on these voters. Khan describes a political party arena of the multiparty political system using nonlinear differential equations with time delay [10]. Their mathematical model gives a comprehensive overview of political competitors. Similarly, Huckfeldt and Kohfeld presents two mathematical models of political parties, one linear and another nonlinear regarding the electoral stability and the decline of class in democratic politics by considering two political parties [9]. In the linear model they have considered behavioural independence within the electorate and in the nonlinear model policy platform, ideology and candidate popularity have been captured. Their model's analysis is a clear picture of politics today.

The current system of democratic election in Nigeria is totally paper-based and manual that takes a lot of time and huge financial expenses [12]. The voters are registered just before the poll so the election commission gets sometimes all the necessary arrangements within this short period of time. INEC added a new voters' register with the previous voters so that the people who have deceased by this time may be considered as the existing voters if they are not informed [13]. So, people may not bestow their faith on the voters' list as it contains numerous fake voters. Again, the authority itself may be corrupted and can allow some fake voters to participate. Voters who reside in foreign countries or miss the registration processes are not considered voters until they inform the authorities, however, in this case, most people do not usually show any interest in the process [14]. Voters may also change place of residence in the period between two elections and if the authority is not informed, they are not considered as voters in that location though constitutionally, they are voters. Therefore, they miss the chance to express this civic right by casting their vote. Sometimes people ruin their votes by stamping on two or more signs mistakenly. This is also a drawback of the paper-based voting system. While casting votes, the acting officers present in the polling unit mark a voter with a black ink on his or her nail but it is removable. So, there is a chance for casting illegal votes. Again, the votes are counted manually, there is a probability that the process becomes a gradual fraught with inaccuracies. These problems necessitated thoughts towards inventing a new system that will reduce corruption and increase accuracy in electoral processes. The concept of the advanced electronic voting system comes from this necessity [15].

2. MATERIALS AND METHODS

2.1. The Proposed Model

In this chapter, we give a detail explanation of model formulation and present model equations. Equilibrium points and stability analysis of the model are discussed. We first present the following assumptions:

I. any opposition party can emerge to be the ruling political party;
II. movement of eligible voters from one political party to another is defection;
III. political party campaigners or godfathers of a given party are its members;
IV. the model ignores votes buying, elections cancellation, declaration of election results inconclusive and antiparty activities;
V. candidate integrity, precedents and political ideology of a party can woo voters confidence;
VI. the class of I, J, E, S and T are not members of any political party.

The influx of voters into the system is determined by the registration rate of voter cards Γ. Hence, voters enter the system at rate Γ into the eligible voter class V. Eligible voters listen to the campaign and are influenced by political party campaigners of a given party G_i, for i = 1, 2, 3. Hence, the rate of political impulse to eligible voters per party is $\phi_i\varphi_i P_i$ where, ϕ_i is the measure of the campaign strategy success of a given political party; P_i and φ_i is the measure of the positive influence of political party campaigners on an eligible voter. Therefore, the waves of political influence on eligible voters is given by $\phi_1\varphi_1 P_1, \phi_2\varphi_2 P_2$, and $\phi_3\varphi_3 P_3$ for the ruling party P_1, major opposition party P_2 and minority parties P_3, respectively. The choice an eligible voter makes is influenced by a voter interest in a given political party P_i and the positive influence of the political party campaigners of the party on the voter. The probability for an eligible voter to accept the ideology of a political party is given by $\phi_i\varphi_i P_i\left(\frac{V}{N}\right)$ where, N is the total population. Thus, an eligible voter enters a political party P_i at a per capita rate $\alpha_i\beta_i$ of where, α_i is interest rate of an eligible voter and β_i is per capita campaign rate of a given party member to an eligible voter that is convinced to join the party. Hence, the rate at which eligible voters become members of a given political party is given by $\alpha_i\beta_i V\left(\frac{P_i}{N}\right)$ where, $\frac{P_i}{N}$ is the probability of an eligible voter coming in contact with a member of a given political party P_i. The model also accounts for the defection between political parties. The per capita defection rate from one political party to another is denoted by γ_{ij}. Thus, the rate at which registered voters defect from party P_i to P_j is given by $\gamma_{ij}P_i\left(\frac{P_j}{N}\right)$ and since there is no defection in the same political party we have $\begin{bmatrix} 0 & \gamma_{12} & \gamma_{13} \\ \gamma_{21} & 0 & \gamma_{23} \\ \gamma_{31} & \gamma_{32} & 0 \end{bmatrix}$ where, $\gamma_{11} = \gamma_{22} = \gamma_{33} = 0$.

The political consultancy rate h_i of a given party P_i to its political party campaigners G_i or the rate at which members of a given party become its political party campaigners is per capita rate given by $(h_i + x_i)P_i G_i$. The model permutes per capital influenced rate q_i of a political party campaigner G_i on its party activities. Therefore, members of a given political party P_i are being influenced by their political party campaigners G_i at the rate $q_i G_i\left(\frac{P_i}{N}\right)$. Modification parameter associated with reduced influence of a political party campaigner G_i and how a political party campaigner developed weak political prowess in the system is also measured in per capita rate of $(1-q_i)\tau_i$ where, $(1-q_i)$ is the reduced influence of a political party campaigner G_i and τ_i is a weak political prowess of such political party campaigner. Hence, the political party campaigners of a given party P_i become politically uninfluential in the space of time at a rate of $(1-q_i)\tau_i G_i$.

In a given political party, P_i we estimate the influences of a political party campaigner G_i to electoral body and thugs in proportions that is enhanced by information, ω_i money θ_i and power η_i. This is a common strategy of political permutation in order to win an election. All the political parties in the system seek to do this through their political party campaigners or political allies. A political campaigner influenced the electoral body and got connected to thugs at a per capita rate of Therefore, the proportion of influence from political $\omega_i\theta_i\eta_i$. party campaigners of a given party P_i to electoral body is I, given by and to $(\omega_i\theta_i\eta_i)G_i$, the thugs T is $(1-\omega_i\theta_i\eta_i)G_i$. Public enlightenment information from electoral body on electoral procedures within the system reaches eligible voters V, security personnel S, electoral observers E, and judiciary at J the rates of $\psi_1 I$, $\psi_2 I$, $\psi_3 I$, and $\psi_4 I$, respectively. The thugs caused chaos during elections that are being fuelled by political influence in the proportions of $\upsilon(1-\rho)T$ to security personnel and the attack rate to the electoral body is ρT. We have assumed retirement rate of the political classes and per capita rate of felony conviction or death to be $(\mu+\sigma)$, this rate is applicable to V, P_i, G_i and T. The classes of I, J, E, and S also reduced through retirement or death rate δ. And the judiciary interprets electoral reforms to electoral body at rate of ψ_5 and ε measured the reduction in compliance from thugs for causing chaos during elections. Figure 1 is a schematic representation of our proposed model.

The solid lines in the diagram represent the influx rates at which individuals enter and leave the system and the rates at which individuals initially choose their political parties. This initial choice depends on how any of the three political groups P_1, P_2, or P_3 convinced an eligible voter. The dashed lines signified the political influence and information within the system. Variables and

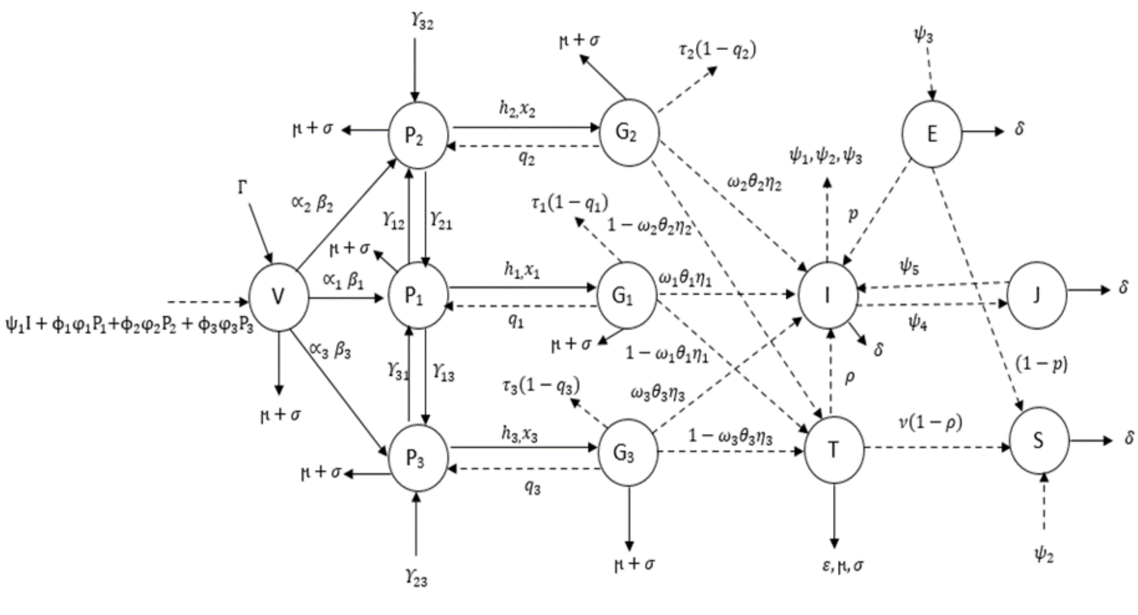

Figure 1. Schematic diagram of the proposed model.

Table 1. Model state variables.

Variable	Description
$V(t)$	Number of eligible voters 18 years and above at time t
$I(t)$	Electoral body at time t
$P_1(t)$	Ruling political party at time t
$P_2(t)$	Major opposition political party at time t
$P_3(t)$	Minority political parties at time t
$G_1(t)$	Political party campaigners of the ruling party
$G_2(t)$	Political party campaigners of opposition party
$G_3(t)$	Political party campaigners of minority parties
$S(t)$	Number of security personnel deployed during election
$E(t)$	Number of electoral observers
$J(t)$	Judiciary/electoral tribunals
$T(t)$	Number of electoral thugs during election

parameters in the model are presented in Tables 1 and 2.

Based on the explanation in section 2.1, we proposed the following model equations:

$$\frac{d\bar{V}}{dt} = \Gamma + \sum_{i=1}^{3}\phi_i\varphi_i\bar{P}_i - \sum_{i=1}^{3}\alpha_i\beta_i\bar{P}_i\frac{\bar{V}}{\bar{N}} - (\mu+\delta)\bar{V} + \psi_1\bar{I} \quad (1)$$

$$\frac{d\bar{P}_1}{dt} = [\alpha_1\beta_1\bar{V} + (\gamma_{21}-\gamma_{12})\bar{P}_2 + (\gamma_{31}-\gamma_{13})\bar{P}_3]\frac{\bar{P}_1}{\bar{N}} - (h_1 + x_1 + \phi_1\varphi_1 + \mu + \sigma)\bar{P}_1 + q_1\bar{G}_1 \quad (2)$$

$$\frac{d\bar{P}_2}{dt} = [\alpha_2\beta_2\bar{V} + (\gamma_{32}-\gamma_{23})\bar{P}_3 + (\gamma_{12}-\gamma_{21})\bar{P}_1]\frac{\bar{P}_2}{\bar{N}} - (h_2 + x_2 + \phi_2\varphi_2 + \mu + \sigma)\bar{P}_2 + q_2\bar{G}_2 \quad (3)$$

$$\frac{d\bar{P}_3}{dt} = [\alpha_3\beta_3\bar{V} + (\gamma_{23}-\gamma_{32})\bar{P}_2 + (\gamma_{13}-\gamma_{31})\bar{P}_1]\frac{\bar{P}_3}{\bar{N}} - (h_3 + x_3 + \phi_3\varphi_3 + \mu + \sigma)\bar{P}_3 + q_3\bar{G}_3 \quad (4)$$

$$\frac{d\bar{G}_1}{dt} = (h_1 + x_1)\bar{P}_1 + [q_1 + (1-q_1)\tau_1 + \omega_1\theta_1\eta_1 + (1-\omega_1\theta_1\eta_1) + \mu + \delta]\bar{G}_1 \quad (5)$$

$$\frac{d\bar{G}_2}{dt} = (h_2 + x_2)\bar{P}_2 + [q_2 + (1-q_2)\tau_2 + \omega_2\theta_2\eta_2 + (1-\omega_2\theta_2\eta_2) + \mu + \delta]\bar{G}_2 \quad (6)$$

$$\frac{d\bar{G}_3}{dt} = (h_3 + x_3)\bar{P}_3 + [q_3 + (1-q_3)\tau_3 + \omega_3\theta_3\eta_3 + (1-\omega_3\theta_3\eta_3) + \mu + \delta]\bar{G}_3 \quad (7)$$

$$\frac{d\bar{I}}{dt} = \omega_1\theta_1\eta_1\bar{G}_1 + \omega_2\theta_2\eta_2\bar{G}_2 + \omega_3\theta_3\eta_3\bar{G}_3 + \mathrm{p}\bar{E} + \rho\bar{T} + \psi_5\bar{J} - (\psi_1+\psi_2+\psi_3+\psi_4+\delta)\bar{I} \quad (8)$$

$$\frac{d\bar{T}}{dt} = (1-\omega_1\theta_1\eta_1)\bar{G}_1 + (1-\omega_2\theta_2\eta_2)\bar{G}_2 + (1-\omega_3\theta_3\eta_3)\bar{G}_3 - (\rho + e_1 + \varepsilon + \mu + \delta)\bar{T} \quad (9)$$

$$\frac{d\bar{S}}{dt} = e_2\bar{E} + \psi_2\bar{I} + e_1\bar{T} - \delta\bar{S} \quad (10)$$

$$\frac{d\bar{E}}{dt} = \psi_3\bar{I} - (p + e_2 + \delta)\bar{E} \quad (11)$$

$$\frac{d\bar{J}}{dt} = \psi_4\bar{I} - (\psi_5 + \delta)\bar{J} \quad (12)$$

$$\bar{N} = \bar{V} + \sum_{i=1}^{3}(\bar{P}_i + \bar{G}_i) + \bar{I} + \bar{T} + \bar{S} + \bar{E} + \bar{J} \quad (13)$$

where $V(0) > 0$, $A(0) > 0$, $B(0) > 0$, $C(0) > 0$, $G_1(0) > 0$, $G_2(0) > 0$, $G_3(0) > 0$, $I(0) > 0$, $T(0) > 0$, $S(0) > 0$, $E(0) > 0$, and $J(0) > 0$.

2.2. Analyses of the Election Model

For the purpose of convenience, we simplified model equations (1) – (12) as follows:

$$\frac{d\bar{V}}{dt} = \Gamma + \sum_{i=1}^{3}d_i\bar{P}_i - [\sum_{i=1}^{3}\Omega_i\bar{P}_i]\frac{\bar{V}}{\bar{N}} - b\bar{V} + \psi_1\bar{I} \quad (14)$$

$$\frac{d\bar{P}_1}{dt} = [\Omega_1\bar{V} - \gamma_1\bar{P}_2 - \gamma_3\bar{P}_3]\frac{\bar{P}_1}{\bar{N}} - k_1\bar{P}_1 + q_1\bar{G}_1 \quad (15)$$

$$\frac{d\bar{P}_2}{dt} = [\Omega_2\bar{V} - \gamma_2\bar{P}_3 - \gamma_1\bar{P}_1]\frac{\bar{P}_2}{\bar{N}} - k_2\bar{P}_2 + q_2\bar{G}_2 \quad (16)$$

$$\frac{d\bar{P}_3}{dt} = [\Omega_3\bar{V} - \gamma_2\bar{P}_2 - \gamma_3\bar{P}_1]\frac{\bar{P}_3}{\bar{N}} - k_3\bar{P}_3 + q_3\bar{G}_3 \quad (17)$$

$$\frac{d\bar{G}_1}{dt} = r_1\bar{P}_1 - k_4\bar{G}_1 \quad (18)$$

$$\frac{d\bar{G}_2}{dt} = r_2\bar{P}_2 - k_5\bar{G}_2 \quad (19)$$

Table 2. Model parameters.

Parameter	Description
Γ	Registration rate of voter cards
α_1	Interest rate of eligible voters to the ruling party
α_2	Interest rate of eligible voters to major opposition party
α_3	Interest rate of eligible voters to minority parties
ϕ_1	Campaign strategy of the ruling party to eligible voters
ϕ_2	Campaign strategy of major opposition party to eligible voters
ϕ_3	Campaign strategy of minority parties to eligible voters
β_1	Per capita campaign rate of the ruling party to eligible voters
β_2	Per capita campaign rate of major opposition party to eligible voters
β_3	Per capita campaign rate of minority parties to eligible voters
φ_1	Influence of political party campaigners of the ruling party to voters
φ_2	Influence of political party campaigners of major opposition party to voters
φ_3	Influence of political party campaigners of minority parties to voters
γ_{12}	Defection rate of the ruling party to major opposition party
γ_{21}	Defection rate of major opposition party to the ruling party
γ_{31}	Defection rate of minority parties to the ruling parties
γ_{13}	Defection rate of the ruling party to minority parties
γ_{23}	Defection rate of major opposition party to minority parties
γ_{32}	Defection rate of minority parties to major opposition party
μ	Retirement rate of political classes which is applicable to $V, P_1, P_2, P_3, G_1, G_2, G_3, T$
σ	Per capita rate of felony conviction or death applicable to $V, P_1, P_2, P_3, G_1, G_2, G_3, T$
h_1	Consultancy rate of the ruling party to its political party campaigners
h_2	Consultancy rate of major opposition party to its political party campaigners
h_3	Consultancy rate of minority parties to their political party campaigners
x_1	Rate at which members of the ruling party become its political party campaigners
x_2	Rate at which members of major opposition party become its political party campaigners
x_3	Rate at which members of minority parties become their political party campaigners
q_1	Influence of the ruling party campaigners to its party activities
q_2	Influence of major opposition party campaigners to its party activities

Table 2. *Cont.*

Parameter	Description
q_3	Influence of minority parties campaigners to their party activities
$(1-q_1)$	Modification parameter associated with reduced influence of political party campaigners of the ruling party
$(1-q_2)$	Modification parameter associated with reduced influence of political party campaigners of major opposition party
$(1-q_3)$	Modification parameter associated with reduced influence of political party campaigners of minority parties
τ_1	Rate at which political party campaigners of the ruling party developed weak political prowess
τ_2	Rate at which political party campaigners of major opposition party developed weak political prowess
τ_3	Rate at which political party campaigners of minority parties developed weak political prowess
$\omega_1\theta_1\eta_1$	Estimates influence of political party campaigners of the ruling party to electoral body and therefore, $(1-\omega_1\theta_1\eta_1)$ goes to thugs which is both enhanced by information, money and power
$\omega_2\theta_2\eta_2$	Estimates influence of political party campaigners of major opposition party to electoral body and therefore, $(1-\omega_2\theta_2\eta_2)$goes to thugs which is both enhanced by information, money and power
$\omega_3\theta_3\eta_3$	Estimates influence of political party campaigners of minority parties to electoral body and therefore, $(1-\omega_3\theta_3\eta_3)$ goes to thugs which is both enhanced by information, money and power
δ	Retirement or death of a member of constituted authorities which is applicable to *I, J, E*, and *S*
υ	Compromising rate of security personnel during elections
$(1-\rho)$	Proportion of thugs attacks to security personnel and therefore, ρ is the proportion of thugs attacks to electoral body during election
ψ_1	Flow of electoral information from electoral body to eligible voters
ψ_2	Compliance rate of security information from electoral body to security personnel during election
ψ_3	Flow of electoral information from electoral body to electoral observers
ψ_4	Flow of electoral information from electoral body to electoral tribunals
$(1-p)$	Proportion of security information from electoral observers to security personnel and therefore, p is the proportion of security information passed to electoral body during election
ψ_5	Modification parameter associated with interpretation of electoral reforms to electoral body by the electoral tribunals
ε	Is measuring the reduction in compliance from thugs to cause chaos during elections

$$\frac{d\bar{G}_3}{dt} = r_3\bar{P}_3 - k_6\bar{G}_3 \quad (20)$$

$$\frac{d\bar{I}}{dt} = a_1\bar{G}_1 + a_3\bar{G}_2 + a_5\bar{G}_3 + \mathrm{p}\bar{E} + \rho\bar{T} + \psi_5\bar{J} - k_7\bar{I} \quad (21)$$

$$\frac{d\bar{T}}{dt} = a_2\bar{G}_1 + a_4\bar{G}_2 + a_6\bar{G}_3 - k_8\bar{T} \quad (22)$$

$$\frac{d\bar{S}}{dt} = e_2\bar{E} + \psi_2\bar{I} + e_1\bar{T} - \delta\bar{S} \quad (23)$$

$$\frac{d\bar{E}}{dt} = \psi_3\bar{I} - k_9\bar{E} \quad (24)$$

$$\frac{d\bar{J}}{dt} = \psi_4\bar{I} - k_{10}\bar{J} \quad (25)$$

where $b = \mu + \delta$; $\gamma_1 = \gamma_{12} - \gamma_{21}$; $\gamma_2 = \gamma_{23} - \gamma_{32}$; $\gamma_3 = \gamma_{13} - \gamma_{31}$; $d_1 = \phi_1\varphi_1$; $d_2 = \phi_2\varphi_2$; $d_3 = \phi_3\varphi_3$; $e_1 = \upsilon(1-\rho)$; $e_2 = 1 - p$; $a_1 = \omega_1\theta_1\eta_1$; $a_2 = 1 - a_1$; $a_3 = \omega_2\theta_2\eta_2$; $a_4 = 1 - a_3$; $a_5 = \omega_3\theta_3\eta_3$; $a_6 = 1 - a_5$; $k_1 = r_1 + d_1 + b$; $k_2 = r_2 + d_2 + b$; $k_3 = r_3 + d_3 + b$; $k_4 = (q_1 + (1-q_1)\tau_1 + a_1 + a_2 + \mathrm{b}$; $k_5 = (q_2 + (1-q_2)\tau_2 + a_3 + a_4 + \mathrm{b}$; $k_6 = (q_3 + (1-q_3)\tau_3 + a_5 + a_6 + \mathrm{b}$; $k_7 = \psi_1 + \psi_2 + \psi_3 + \psi_4 + \delta$; $k_8 = \rho + e_1 + \varepsilon + \mathrm{b}$; $k_9 = p + e_2 + \delta$; $\Omega_1 = \alpha_1\beta_1$; $\Omega_2 = \alpha_2\beta_2$; $\Omega_3 = \alpha_3\beta_3$; $r_1 = h_1 + x_1$; $r_2 = h_2 + x_2$; $r_3 = h_3 + x_3$; $k_{10} = \psi_5 + \delta$ & $\mathrm{i} = 1, 2, 3$.

2.3. Stability Analysis of the Election Model

In this subsection, we discussed the stability analysis of the election model by introducing the following new state variables:

$V = \frac{\bar{V}}{\bar{N}}$; $P_i = \frac{\bar{P}_i}{\bar{N}}$; $G_i = \frac{\bar{G}_i}{\bar{N}}$; $I = \frac{\bar{I}}{\bar{N}}$; $T = \frac{\bar{T}}{\bar{N}}$; $S = \frac{\bar{S}}{\bar{N}}$; $E = \frac{\bar{E}}{\bar{N}}$; $J = \frac{\bar{J}}{\bar{N}}$ and $N = \frac{b\bar{N}}{\Gamma}$, to scale each subpopulation as a proportion of the total population N. Hence, $V + P_1 + P_2 + P_3 + sdG_1 + G_2 + G_3 + I + T + S + E + J = 1$ Models (1) to (12) become,

$$\frac{dV}{dt^*} = 1 + \sum_{i=1}^{3} d_i^* P_i + \psi_1^*\mathrm{I} - \sum_{i=1}^{3} \Omega_i^* P_i V - V - \frac{V}{N}\frac{dN}{dt^*} \quad (26)$$

$$\frac{dP_1}{dt^*} = [\Omega_1^* V - \gamma_1^* P_2 - \gamma_3^* P_3]P_1 - k_1^* P_1 + q_1^* G_1 - \frac{P_1}{N}\frac{dN}{dt^*} \quad (27)$$

$$\frac{dP_2}{dt^*} = [\Omega_2^* V - \gamma_2^* P_3 - \gamma_1^* P_1]P_2 - k_2^* P_2 + q_2^* G_2 - \frac{P_2}{N}\frac{dN}{dt^*} \quad (28)$$

$$\frac{dP_3}{dt^*} = [\Omega_3^* V - \gamma_2^* P_2 - \gamma_3^* P_1]P_3 - k_3^* P_3 + q_3^* G_3 - \frac{P_3}{N}\frac{dN}{dt^*} \quad (29)$$

$$\frac{dG_1}{dt^*} = r_1^* P_1 - k_4^* G_1 - \frac{G_1}{N}\frac{dN}{dt^*} \quad (30)$$

$$\frac{dG_2}{dt^*} = r_2^* P_2 - k_5^* G_2 - \frac{G_2}{N}\frac{dN}{dt^*} \quad (31)$$

$$\frac{dG_3}{dt^*} = r_3^* P_3 - k_6^* G_3 - \frac{G_3}{N}\frac{dN}{dt^*} \quad (32)$$

$$\frac{dI}{dt^*} = a_1^* G_1 - a_3^* G_2 + a_5^* G_3 + P^*\mathrm{E} + \rho^*\mathrm{T} + \psi_5^* J - k_7^* I - \frac{I}{N}\frac{dN}{dt^*} \quad (33)$$

$$\frac{dT}{dt^*} = a_2^* G_1 - a_4^* G_2 + a_6^* G_3 - k_8^*\mathrm{T} - \frac{T}{N}\frac{dN}{dt^*} \quad (34)$$

$$\frac{dS}{dt^*} = e_2^*\mathrm{E} + \psi_2^*\mathrm{I} + a_6^* G_3 - e_1^*\mathrm{T} - \delta^*\mathrm{S} - \frac{S}{N}\frac{dN}{dt^*} \quad (35)$$

$$\frac{dE}{dt^*} = \psi_3^*\mathrm{I} - k_9^*\mathrm{E} - \frac{E}{N}\frac{dN}{dt^*} \quad (36)$$

$$\frac{dJ}{dt^*} = \psi_4^*\mathrm{I} - k_{10}^*\mathrm{J} - \frac{J}{N}\frac{dN}{dt^*} \quad (37)$$

$$\frac{dN}{dt^*} = [1 + (\delta^* - 1)(\mathrm{I} + \mathrm{J} + \mathrm{S} + \mathrm{E}) + \sum_{i=1}^{3}(1 - q_i)\tau_i^*\bar{G}_i + \varepsilon^*\mathrm{T}]\mathrm{N} \quad (38)$$

where $t^* = \mathrm{bt}$; $k_i^* = \frac{k_i}{b}$

For $i = 1, \ldots, 10$; $r_i^* = \frac{r_i}{b}$; $d_i^* = \frac{b_i}{b}$; $\Omega_i^* = \frac{\Omega_i}{b}$.

For $i = 1, \ldots 3$; $a_i^* = \frac{a_i}{b}$

For $i = 1, \ldots 6$; $e_i^* = \frac{e_i}{b}$

For $i = 1, \ldots 2$; $\delta_i^* = \frac{\delta_i}{b}$ and $\tau_i^* = \frac{\tau_i}{b}$

For $i = 1, \ldots 3$ dropping the asterisk notation and since

$$V = 1 - \sum_{i=1}^{3}(P_i + G_i) + \mathrm{I} + \mathrm{T} + \mathrm{S} + \mathrm{E} + \mathrm{J} \quad (39)$$

$$\frac{dP_1}{dt} = [\Omega_1 V - \gamma_1 P_2 - \gamma_3 P_3]P_1 - k_1 P_1 + q_1 G_1 - \frac{P_1}{N}\frac{dN}{dt} \quad (40)$$

$$\frac{dP_2}{dt} = [\Omega_2 V - \gamma_2 P_3 - \gamma_1 P_1]P_2 - k_2 P_2 + q_2 G_2 - \frac{P_2}{N}\frac{dN}{dt} \quad (41)$$

$$\frac{dP_3}{dt} = [\Omega_3 V - \gamma_2 P_2 - \gamma_3 P_1]P_3 - k_3 P_3 + q_3 G_3 - \frac{P_3}{N}\frac{dN}{dt} \quad (42)$$

$$\frac{dG_1}{dt} = r_1 P_1 - k_4 G_1 - \frac{G_1}{N}\frac{dN}{dt} \quad (43)$$

$$\frac{dG_2}{dt} = r_2 P_2 - k_5 G_2 - \frac{G_2}{N}\frac{dN}{dt} \quad (44)$$

$$\frac{dG_3}{dt} = r_3 P_3 - k_6 G_3 - \frac{G_3}{N}\frac{dN}{dt} \quad (45)$$

$$\frac{dI}{dt} = a_1 G_1 - a_3 G_2 + a_5 G_3 + \mathrm{pE} + \rho\mathrm{T} + \psi_5 J - k_7 I - \frac{I}{N}\frac{dN}{dt} \quad (46)$$

$$\frac{dT}{dt} = a_2 G_1 - a_4 G_2 + a_6 G_3 - k_8\mathrm{T} - \frac{T}{N}\frac{dN}{dt} \quad (47)$$

$$\frac{dS}{dt} = e_2\mathrm{E} + \psi_2\mathrm{I} + a_6 G_3 - e_1\mathrm{T} - \delta\mathrm{S} - \frac{S}{N}\frac{dN}{dt} \quad (48)$$

$$\frac{dE}{dt} = \psi_3\mathrm{I} - k_9\mathrm{E} - \frac{E}{N}\frac{dN}{dt} \quad (49)$$

$$\frac{dJ}{dt} = \psi_4 \mathrm{I} - k_{10}\mathrm{J} - \frac{J}{N}\frac{dN}{dt} \quad \textbf{(50)}$$

$$\frac{dN}{dt} = 1 - [1 + (\delta - 1)(\mathrm{I} + \mathrm{J} + \mathrm{S} + \mathrm{E}) + \sum_{i=1}^{3}(1 - q_i)\tau_i G_i - \varepsilon\, \mathrm{T}]\mathrm{N} \quad \textbf{(51)}$$

2.4. Equilibrium Points of Election Model

In this subsection, we discussed the equilibrium points of election model. At the equilibrium points,

$$\frac{dN}{dt} = \frac{dP_1}{dt} = \frac{dP_2}{dt} = \frac{dP_3}{dt} = \frac{dG_1}{dt} = \frac{dG_2}{dt} = \frac{dG_3}{dt} = \frac{dI}{dt} = \frac{dT}{dt} = \frac{dS}{dt} = \frac{dE}{dt} = 0$$

Thus, equations (40) – (51) becomes

$$[\Omega_1 V - \gamma_1 P_2 - \gamma_3 P_3]P_1 - k_1 P_1 + q_1 G_1 = 0 \quad \textbf{(52)}$$
$$[\Omega_2 V - \gamma_2 P_3 - \gamma_1 P_1]P_2 - k_2 P_2 + q_2 G_2 = 0 \quad \textbf{(53)}$$
$$[\Omega_3 V - \gamma_2 P_2 - \gamma_3 P_1]P_3 - k_3 P_3 + q_3 G_3 = 0 \quad \textbf{(54)}$$
$$r_1 P_1 - k_4 G_1 = 0 \quad \textbf{(55)}$$
$$r_2 P_2 - k_5 G_2 = 0 \quad \textbf{(56)}$$
$$r_3 P_3 - k_6 G_3 = 0 \quad \textbf{(57)}$$
$$a_1 G_1 - a_3 G_2 + a_5 G_3 + \mathrm{pE} + \rho\mathrm{T} + \psi_5 J - k_7 I = 0 \quad \textbf{(58)}$$
$$a_2 G_1 - a_4 G_2 + a_6 G_3 - k_8 \mathrm{T} = 0 \quad \textbf{(59)}$$
$$e_2 \mathrm{E} + \psi_2 \mathrm{I} + a_6 G_3 - e_1 \mathrm{T} - \delta \mathrm{S} = 0 \quad \textbf{(60)}$$
$$\psi_3 \mathrm{I} - k_9 \mathrm{E} = 0 \quad \textbf{(61)}$$
$$\psi_4 \mathrm{I} - k_{10} \mathrm{J} = 0 \quad \textbf{(62)}$$
$$1 - [1 + (\delta - 1)(\mathrm{I} + \mathrm{J} + \mathrm{S} + \mathrm{E}) + \sum_{i=1}^{3}(1 - q_i)\tau_i G_i - \varepsilon\, \mathrm{T}]\mathrm{N} = 0 \quad \textbf{(63)}$$

Solve (55) – (62) in terms of P_1, P_2 and P_3 to get

$$G_1 = \frac{r_1 P_1}{k_4} \quad \textbf{(64)}$$

$$G_2 = \frac{r_2 P_2}{k_5} \quad \textbf{(65)}$$

$$G_3 = \frac{r_3 P_3}{k_6} \quad \textbf{(66)}$$

$$\mathrm{T} = \frac{A_0 P_1 + B_o P_2 + C_o P_3}{X_o} \quad \textbf{(67)}$$

$$\mathrm{I} = \frac{A_1 P_1 + B_1 P_2 + C_1 P_3}{X_1} \quad \textbf{(68)}$$

$$\mathrm{S} = \frac{A_2 P_1 + B_2 P_2 + C_2 P_3}{X_1} \quad \textbf{(69)}$$

$$\mathrm{E} = \frac{\psi_2 I}{k_9} \quad \textbf{(70)}$$

$$\mathrm{J} = \frac{\psi_4 I}{k_{10}} \quad \textbf{(71)}$$

where $A_o = r_1 a_2 k_5 k_6$, $B_o = r_2 a_4 k_4 k_6$, $C_o = r_3 a_6 k_4 k_5$, $A_1 = r_1 k_5 k_6 k_7 k_9 (a_1 k_8 + a_2)$, $B_1 = r_2 k_4 k_5 k_7 k_9 (a_3 k_8 + a_4)$, $C_1 = r_3 k_4 k_6 k_7 k_9 (a_5 k_8 + a_6)$, $X_o = k_4 k_5 k_6 k_8 X_1 = X_o(k_7 k_9 - \rho\psi_3 - \psi_4\psi_5)$, $A_2 = A_1 X_o(e_2\psi_3 + \psi_1 k_9) + A_0 X_1 e_1 k_9$, $B_2 = B_1 X_o(e_2\psi_3 + \psi_2 k_9) + B_0 X_1 e_1 k_9$,

$C_2 = C_1 X_o(e_2\psi_3 + \psi_2 k_9) + C_0 X_1 e_1 k_9$, and $X_2 = X_o X_1 k_9 \delta$

Substitute (64) – (66) in to (39) to have

$$V = 1 - Z_1 P_1 - Z_2 P_2 - Z_3 P_3 \quad \textbf{(72)}$$

where

$$A_3 = k_4 k_5 k_6 X_o X_2 (k_9 k_{10} + k_9 \psi_4 + k_{10}\psi_3) A_1 + k_5 k_6 k_9 k_{10} X_1 (A_o X_2 + A_2 X_o) + k_5 k_6 k_9 k_{10} X_o X_1 X_2 (k_4 + r_1),$$

$$B_3 = k_4 k_5 k_6 X_o X_2 (k_9 k_{10} + k_9 \psi_4 + k_{10}\psi_3) B_1 + k_5 k_6 k_9 k_{10} X_1 (B_o X_2 + B_2 X_o) + k_4 k_6 k_9 k_{10} X_o X_1 X_2 (k_5 + r_2),$$

$$C_3 = k_4 k_5 k_6 X_o X_2 (k_9 k_{10} + k_9 \psi_4 + k_{10}\psi_3) C_1 + k_5 k_6 k_9 k_{10} X_1 (C_o X_2 + C_2 X_o) + k_4 k_5 k_9 k_{10} X_o X_1 X_2 (k_6 + r_3),$$

$$Z_1 = \frac{A_3}{X_3},\ Z_2 = \frac{B_3}{X_3} Z_3 = \frac{C_3}{X_3}$$

Substitute (64) – (66) and (72) in to (52) – (54) to obtain

$$P_1[\Omega_1(1 - Z_1 P_1 - Z_2 P_2 - Z_3 P_3) - \gamma_1 P_2 - \gamma_3 P_3 - k_1 + R_1] = 0 \quad \textbf{(73)}$$
$$P_2[\Omega_2(1 - Z_1 P_1 - Z_2 P_2 - Z_3 P_3) - \gamma_2 P_3 + \gamma_1 P_1 - k_2 + R_2] = 0 \quad \textbf{(74)}$$
$$P_3[\Omega_3(1 - Z_1 P_1 - Z_2 P_2 - Z_3 P_3) + \gamma_2 P_2 + \gamma_3 P_1 - k_3 + R_3] = 0 \quad \textbf{(75)}$$

Respectively such that $R_1 = \frac{q_1 r_1}{k_4} R_2 = \frac{q_2 r_2}{k_5}$ and $R_3 = \frac{q_3 r_3}{k_6}$

The solution of (73) to (75) will give eight equilibrium points but due to the following facts that

i. P_2 exist (that is $P_2 \neq 0$) only when P_1 exist (that is $P_1 \neq 0$)
ii. P_3 exist (that is $P_3 \neq 0$) only when P_2 exist (that is $P_2 \neq 0$)

Thus, is logically reasonable to consider the following equilibrium points.

a. **Party-free equilibrium**: Let Q_o denote the party free equilibrium which corresponds to the absence of political parties. This means that Q_o

should satisfy

$P_1^0 = P_2^0 = P_3^0 \quad = G_1^0 = G_2^0 = G_3^0 \quad = I^o = T^o \quad = S^o = E^o = J^o = 0$ and $N^o = 1$ **(76)**

The solution of (73) - (75) gives $P_1^0 = P_2^0 = P_3^0 = 0$ which is then substituted to (63) to validate (76). Hence Q_o exist unconditionally and nonpartisan system.

b. **Single party equilibrium:** The single party system is the system that legally recognizes the ruling party. This means that only the ruling party is permitted to exist ($P_1^* \neq 0$).

Let Q_1 denotes the equilibrium in a dictatorship system. Thus $P_1^* > 0$ such that $P_2^* = P_3^* = 0$ (79) becomes

$$P_1[\Omega_1(1 - Z_1P_1-) - k_1 + R_1] = 0 \quad \textbf{(77)}$$

which implies

$$P_1^* = \frac{\Omega_1 + R_1 - k_1}{\Omega_1 Z_1} \quad \textbf{(78)}$$

The single party equilibrium exists if any of the conditions below is satisfied

i. $\Omega_1 + R_1 > k_1$
ii. $\Omega_1 > k_1$
iii. $R_1 > k_1$

c. **Dual party system.** For model (1) – (12), the dual party system exist in the absence of the minority party that is ($P_3^* = 0$). It means that the State officially permits only two political parties that is $P_1^* \neq 0$ and $P_2^* \neq 0$.

Let Q_2 denotes the dual party equilibrium such that (73) to (74) becomes

$$\Omega_1(1 - Z_1P_1^* - Z_2P_2^*) - \gamma_1 P_2^* - k_1 + R_1 = 0 \quad \textbf{(79)}$$

$$\Omega_2(1 - Z_1P_1^* - Z_2P_2^*) - \gamma_1 P_2^* - k_2 + R_2 = 0 \quad \textbf{(80)}$$

Solve (79) and (80) simultaneously to obtain

$$P_1^* = \frac{(\Omega_1 Z_2 + \gamma_1)(k_2 - R_2) - \Omega_2[\gamma_1 + Z_2(k_1 - R_1)]}{\gamma_1(\gamma_1 + \Omega_1 Z_2 - \Omega_2 Z_1)} \quad \textbf{(81)}$$

$$P_2^* = \frac{\Omega_1[\gamma_1 + Z_1(k_2 - R_2)] - (k_1 - R_1)(\gamma_1 - \Omega_2 Z_1)}{\gamma_1(\gamma_1 + \Omega_1 Z_2 - \Omega_2 Z_1)} \quad \textbf{(82)}$$

respectively. Hence, the dual party equilibrium that is $P_1^* > 0$, $P_2^* > 0$ and $P_3^* = 0$ exist provided the model's parameters satisfice the following conditions

$$\Omega_1 Z_2 > \Omega_2 Z_1, \quad \textbf{(83)}$$

$$(\Omega_1 Z_2 + \gamma_1)(k_2 - R_2) - \Omega_2[\gamma_1 + Z_2(k_1 - R_1) > 0 \quad \textbf{(84)}$$

$$\Omega_1[\gamma_1 + Z_1(k_2 - R_2)] - (k_1 - R_1)(\gamma_1 - \Omega_2 Z_1) > 0 \quad \textbf{(85)}$$

$$P_2^* = 0 \quad \textbf{(86)}$$

The components of Q_2 are easily gotten by substituting (87) and (88) and into (86)-(77)

d. **Three-party Equilibrium:** The coexistence of the ruling, opposition and minority political parties that is ($P_1^+ \neq 0$, $P_1^+ \neq 0$ and $P_1^+ \neq 0$) implies that the three-party exist. This further means that (73) – (75) becomes

$$\Omega_1(1 - Z_1P_1^+ - Z_2P_2^+ - Z_3P_3^+) - \gamma_3 P_3^+ - \gamma_1 P_2^+ - k_1 + R_1 = 0 \quad \textbf{(87)}$$

$$\Omega_2(1 - Z_1P_1^+ - Z_2P_2^+ - Z_3P_3^+) - \gamma_2 P_3^+ - \gamma_1 P_1^+ - k_2 + R_2 = 0 \quad \textbf{(88)}$$

$$\Omega_3(1 - Z_1P_1^+ - Z_2P_2^+ - Z_3P_3^+) + \gamma_3 P_1^+ + \gamma_2 P_2 - k_3 + R_3 = 0 \quad \textbf{(89)}$$

Solution of (87) – (89) gives

$$P_1^+ = \frac{y_1 - y_2}{y_3} \quad \textbf{(90)}$$

$$P_2^+ = \frac{y_4 - y_5}{y_3} \quad \textbf{(91)}$$

$$P_3^+ = \frac{y_6 - y_7}{y_3} \quad \textbf{(92)}$$

where $f_1 = \Omega_1 Z_1$; $f_2 = \gamma_1 + \Omega_1 Z_2$; $f_3 = \gamma_3 + \Omega_1 Z_3 Z_4 = k_1 - \Omega_1 - R_1$; $f_4 = \gamma_1 - \Omega_2 Z_1$; $f_5 = \Omega_2 Z_2; and\ f_6 = \gamma_2 + \Omega_2 Z_3 Z_5 = k_2 - \Omega_2 - R_2$

$y_1 = f_2 f_6 Z_6 + f_5 f_9 Z_4 + f_8 f_3 Z_5 y_2 = f_2 f_9 Z_5 + f_5 f_3 Z_6 + f_8 f_6 Z_4$
$y_3 = f_7 f_2 Z_6 - [f_1(f_5 f_9 + f_8 f_6) + f_4(f_2 f_9 + f_8 f_3) + f_7 f_2 f_3],$
$f_7 = \gamma_3 - \Omega_3 Z_1 f_8 = \gamma_2 - \Omega_3 Z_z f_8 = \Omega_3 Z_3$
$y_4 = f_1 f_9 Z_5 + f_4 f_9 Z_4 + f_7 f_3 Z_4 y_5 = f_1 f_6 Z_6 + f_7 f_4 Z_4$
and $y_7 = f_2 f_7 Z_5$

Therefore, the three-party equilibrium exists if the parameters of the model satisfy the following condition

i. $y_3 > 0$
ii. $y_1 > y_2$
iii. $y_4 > y_5$
iv. $y_6 > y_7$

The other components of the three-party equilibrium are easily gotten by substituting (90)-(92) into (64)-(71).

$$
\begin{bmatrix}
\Omega_1 - K_1 & 0 & 0 & q_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \Omega_2 - K_2 & 0 & 0 & q_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \Omega_3 - K_3 & 0 & 0 & q_3 & 0 & 0 & 0 & 0 & 0 & 0 \\
r_1 & 0 & 0 & -K_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & r_2 & 0 & 0 & -K_5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & r_3 & 0 & 0 & -K_6 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & a_1 & a_3 & a_5 & -K_7 & \rho & 0 & p & \psi_5 & 0 \\
0 & 0 & 0 & a_2 & a_4 & a_6 & 0 & -K_8 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \psi_2 & e_1 & -\delta & e_2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \psi_3 & 0 & 0 & -K_9 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \psi_4 & 0 & 0 & 0 & -K_{10} & 0 \\
0 & 0 & 0 & -(1-q_1)\tau_1 & -(1-q_2)\tau_2 & -(1-q_3)\tau_3 & -g & -\varepsilon & -g & -g & -g & -1
\end{bmatrix}
$$

2.5. Local Stability of Party-free Equilibrium

In this subsection, we discussed the local stability of party-free equilibrium using the Jacobian matrix on the model equations (1) to (13).

The characteristic equation of the above is simplified with the aid of Maple 18 software as

$$(\lambda + 1)(\lambda + \delta)(\lambda + k_8)W_1W_2W_3W_4 = 0 \qquad \textbf{(93)}$$

where

$$W_1 = \lambda^2 + (k_1 + k_4 - \Omega_1)\lambda + k_1k_4 - \Omega_1k_4 - q_1r_1 \qquad \textbf{(94)}$$

$$W_2 = \lambda^2 + (k_2 + k_5 - \Omega_2)\lambda + k_2k_5 - \Omega_2k_5 - q_2r_2 \qquad \textbf{(95)}$$

$$W_3 = \lambda^2 + (k_3 + k_6 - \Omega_3)\lambda + k_6(k_3 - \Omega_3) - q_3r_3 \qquad \textbf{(96)}$$

$$W_4 = \lambda^3 + (k_7 + k_9 - k_{10})\lambda^2 + (k_7k_9 + k_7k_{10} + k_9k_{10} - \psi_4\psi_5 - \rho\psi_3)\lambda + k_7k_9k_{10} - k_9\psi_4\psi_5 - k_{10}\psi_3 \qquad \textbf{(97)}$$

It obvious from (93) that $\lambda_1 = -1 < 0, \lambda_2 = -\delta < 0$, $\lambda_3 = -k_8 < 0$

Using Routh- Hurwitz criterion for the characteristic equation of the form

$$\lambda^n + a_{n-1}\lambda^{n-1} + \cdots + a_1\lambda + a_0 = 0 \qquad \textbf{(98)}$$

where

λ is the eigenvalue of (98) and n is the degree of (98)

a_i is the coefficient of (98) such that $0 = i < n - 1$

Then, (98) is locally asymptotically stable for

i. $n = 2$ if and only if $a_i > 0$
ii. $n = 3$ if and only if $a_i > 0$ and $a_1a_2 > a_0$

Thus, (94) is stable if and $k_1 > \Omega_1$ $k_4(k_1 - \Omega_1) - q_1r_1 > 0$

Similarly, (95) – (96) are stable if $k_2 > \Omega_2$, $k_5(k_2 - \Omega_2) - q_2r_2 > 0, k_3 > \Omega_3$ and $k_6(k_3 - \Omega_3) - q_3r_3 > 0$ respectively.

Finally, W_4 is stable since all its coefficients are positive.

In summary, the party-free equilibrium is locally asymptotically stable if and only if the following conditions are satisfied, otherwise unstable.

i. $k_1 > \Omega_1$, $k_4(k_1 - \Omega_1) - q_1r_1 > 0$
ii. $k_2 > \Omega_2$, $k_5(k_2 - \Omega_2) - q_2r_2 > 0,$
iii. $k_3 > \Omega_3$, $k_6(k_3 - \Omega_3) - q_3r_3 > 0$

2.6. Boundedness of the Election Model

In this subsection, we discussed the boundedness of the election model and show the feasible region of the model. We begin by differentiating (13) with respect to time t, to have

$$\frac{d\bar{N}}{dt} = \frac{d\bar{V}}{dt} + \sum_{i=1}^{3}\frac{d}{dt}(\bar{P}_i + \bar{G}_i) + \frac{d\bar{I}}{dt} + \frac{d\bar{T}}{dt} + \frac{d\bar{S}}{dt} + \frac{d\bar{E}}{dt} + \frac{d\bar{J}}{dt} \qquad \textbf{(99)}$$

Substitute (14) – (25) in to (99) with further simplification to get

$$\frac{d\bar{N}}{dt} = \Gamma - b[\bar{V} + \sum_{i=1}^{3}(\bar{P}_i + \bar{G}_i) + \bar{T}] - \varepsilon\bar{T} - \sum_{i=1}^{3}(1 - q_i)\tau_i\bar{G}_i - \delta(\bar{I} + \bar{S} + \bar{E} + \bar{J}) \qquad \textbf{(100)}$$

It is obvious from (13) to note

$$\bar{N} - \bar{I} + \bar{S} + \bar{E} + \bar{J} = \bar{V} + \sum_{i=1}^{3}(\bar{P}_i + \bar{G}_i) + \bar{T} \qquad \textbf{(101)}$$

Thus, (100) becomes

$$\frac{d\bar{N}}{dt} = \Gamma - \mathrm{b}\bar{N} + (b-\delta)(\bar{I} + \bar{S} + \bar{E} + \bar{J}) - \sum_{i=1}^{3}(1-q_i)\tau_i\bar{G}_i - \varepsilon\,\bar{T} \quad \textbf{(102)}$$

Lemma 1: The feasibility region D defined by $D = \{(V, P_1, P_2, P_3, G_1, G_2, G_3, I, T, S, E, J)\epsilon R_+^{12}\ ,\ N \leq \frac{\Gamma}{b}\ \}$, with non-negative initial condition are positive invariant for model (1) to (12) if $\delta > b$

Proof

From (102)

$$\frac{d\bar{N}}{dt} \leq \Gamma - \mathrm{b}\bar{N} \ \text{ if } \ \delta > b \quad \textbf{(103)}$$

Applying Birkhoff's and Rota's theorem on (30) to get

$$N(t) \leq \frac{\Gamma}{b} + e^{-bt}(N_0 - \frac{\Gamma}{b}) \quad \textbf{(104)}$$

As $t \to \infty$, (104) becomes $0 \leq N(t) \leq \frac{\Gamma}{b}$. This shows that the model can be studied in the feasible region,

$$D = \{(V, P_1, P_2, P_3, G_1, G_2, G_3, I, T, S, E, J)\epsilon R_+^{12}, N \leq \frac{\Gamma}{b}\}.$$

Table 3. The following model parameters serve as inputs into election model.

Parameters	Values	Ref	Parameters	Values	Ref
Γ	0.5	inec	x_3	0.5	estimated
α_1	0.7	Plausible value	q_1	0.9	Plausible value
α_2	0.5	Plausible value	q_2	0.6	Plausible value
α_3	0.2	Plausible value	q_3	0.5	Plausible value
ϕ_1	0.3	estimated	τ_1	0.3	Plausible value
ϕ_2	0.2	estimated	τ_2	0.4	Plausible value
ϕ_3	0.1	estimated	τ_3	0.5	Plausible value
β_1	0.9	estimated	ω_1	0.6	Plausible value
β_2	0.7	estimated	ω_2	0.4	Plausible value
β_3	0.5	estimated	ω_3	0.2	Plausible value
φ_1	0.3	estimated	θ_1	0.6	Plausible value
φ_2	0.2	estimated	θ_2	0.4	Plausible value
φ_3	0.1	estimated	θ_3	0.2	Plausible value
γ_{12}	0.1	estimated	η_1	0.5	Plausible value
γ_{21}	0.4	estimated	η_2	0.4	Plausible value
γ_{31}	0.6	estimated	η_3	0.3	Plausible value
γ_{13}	0.1	estimated	δ	0.1	inec
γ_{23}	0.1	estimated	ν	0.7	estimated
γ_{32}	0.2	estimated	$1-\rho$	0.6	Plausible value
μ	0.2	estimated	ψ_1	0.7	inec
σ	0.1	estimated	ψ_2	0.5	inec
h_1	0.8	estimated	ψ_3	0.7	inec
h_2	0.5	estimated	ψ_4	0.8	inec
h_3	0.1	estimated	$1-p$	0.8	Plausible value
x_1	0.5	estimated	ψ_5	0.1	estimated
x_2	0.5	estimated	ε	0.2	inec

Table 4. Results of presidential election 2007-2019 (number of votes obtained by candidates).

Status	2007	2011	2015	2019	Ref
Ruling Party	24638063	22495187	12853162	11262978	INEC
Major opposition party	6605299	12214853	15424921	15191847	INEC
Minority opposition parties	4045622	3499938	309481	869758	INEC

Table 5. Results of presidential election 2007-2019 by percentage of valid votes.

Status	2007	2011	2015	2019	Ref
Ruling Party	69.60%	58.87%	44.96%	41.22%	INEC
Major opposition party	18.66%	31.97%	53.96%	55.60%	INEC
Minority opposition parties	11.74%	9.16%	1.08%	3.18%	INEC

Table 6. Results of presidential election by state from 2007-2019.

Status	2007	2011	2015	2019	Ref
Ruling Party	33	25	16	18	INEC
Major opposition party	4	11	21	19	INEC
Minority opposition parties	0	1	0	0	INEC

It can be verified that D is positively invariant with respect to the model equations (1) – (12).

3. RESULTS AND DISCUSSIONS

3.1. Sensitivity Analysis, Numerical Simulations and Curve Fittings

In this section, we present results for research findings and discussions as follows: analysis of Presidential, Senate Seats, House of Representatives, and Governorship elections over the years. We first invoked the application of curve fitting and numerical simulation to examine the relationship between one or more predictors to a response variable, with the goal of defining a "best fit" relationship within the model variables and parameters. A linear model of poly degree three is used to checkmate the trajectories of each political party.

3.2. Case 1-Analysis of Presidential Elections over the Years

Figures 2 and 3 show that the number of votes obtained by the ruling party has been declining over the years, as indicated by the downward trend in the curve of Figure 2. The ruling party obtained the highest number of votes in the 2007 election, with over 24 million votes, but has been obtaining fewer votes in each subsequent election. The major opposition party, on the other hand, has been gaining ground over the years, as indicated by the upward trend in the curve. The major opposition party obtained the lowest number of votes in the 2007 election, with just over 11 million votes, but has been obtaining more votes in each subsequent election. Minority opposition parties have not been able to pose a significant challenge to the ruling party and the major opposition party, as indicated by the relatively flat curve.

Figure 4 indicates that the performance of the major opposition party has been increasing steadily over the years, as indicated by the upward trend in the curve. The curve suggests that the major opposition party's performance was at its lowest in the 2007 election and has been increasing steadily since then.

Figure 5 indicates that the performance of the ruling party has been declining over the years, as indicated by the downward trend in the curve. The curve suggests that the ruling party's performance peaked in the 2007 election and has been declining steadily. Curve fitting indicates that the performance of the ruling party is influenced by several factors, including the popularity of the ruling party's policies, the performance of the economy, and the level of public trust in the ruling party.

Figure 6 shows the performance of minority opposition parties, is relatively low and inconsistent over the years, as indicated by the flat curve. The linear model used to fit the curve indicates that the performance of minority opposition parties is influenced by several factors, including the popularity of their policies, the level of public trust in them, and their ability to form strong alliances within the structure of the parties.

3.3. Case 2-Analysis of Senate Seats Won at the National Assembly over the Years

Figures 7 and 8 are visual representations of the performance of the ruling party, the major opposition party, and minority opposition parties in senate elections over the years. The performance of the ruling party has been fluctuating over the years, with some years having lower performance than others. For example, the ruling party won 85 seats in 2007, but only 45 seats in 2019. This suggests that the ruling party has faced challenges in maintaining its dominance at the Senate, and that its performance has been affected by various factors such as changes in leadership, policy decisions, and public opinion.

Overall, the graph of Figure 9 shows that the performance of the ruling party at the Senate has been fluctuating over the years, with some years having higher performance than others. The fitted curve provides a good representation of the trend in the performance of the ruling party, with a high goodness of fit (R-square of 1).

The performance of the major opposition party at the Senate is depicted in Figure 10. The graph shows a fitted curve that represents a linear model similar to the performance of the ruling party, with coefficients p1, p2, p3, and p4. The coefficients are

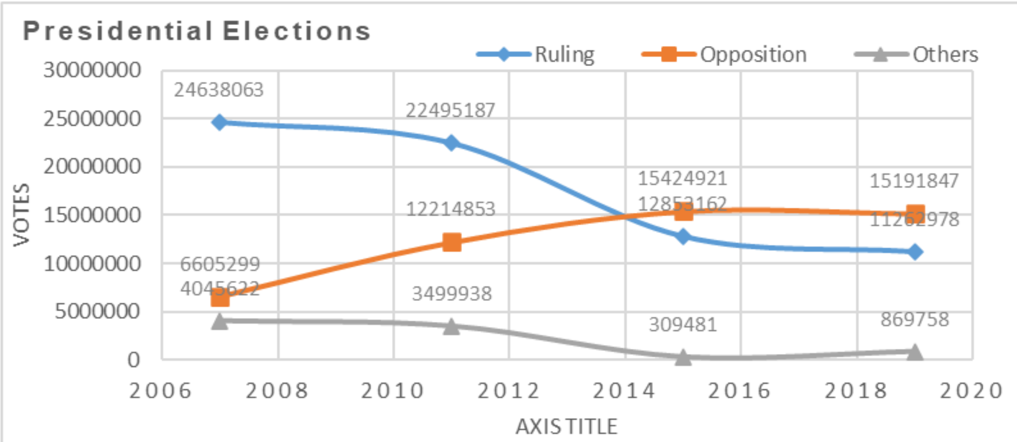

Figure 2. Performance plots of parties in presidential elections.

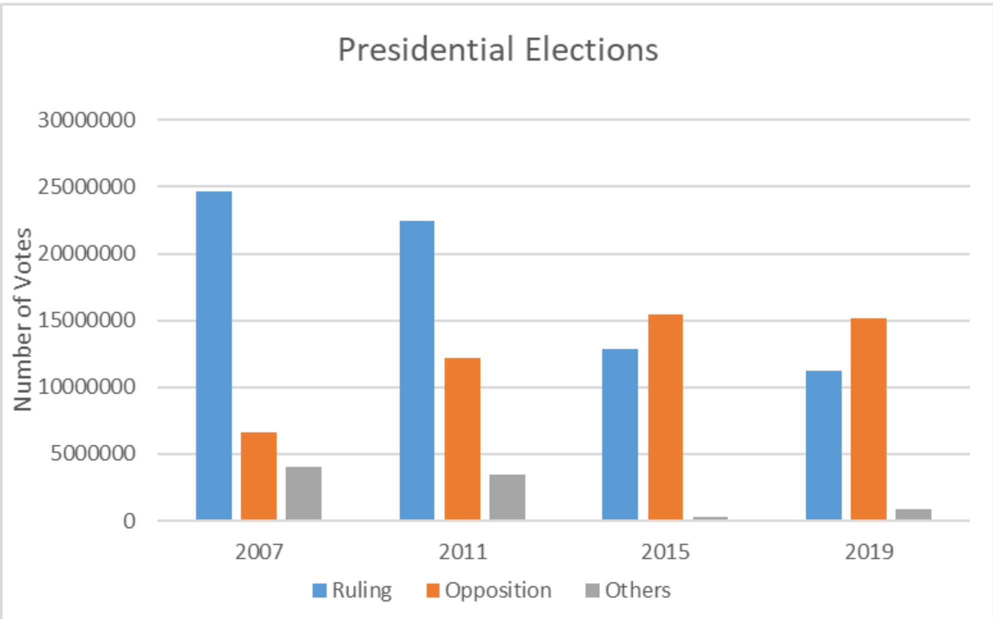

Figure 3. Bar chart representing political parties in presidential elections.

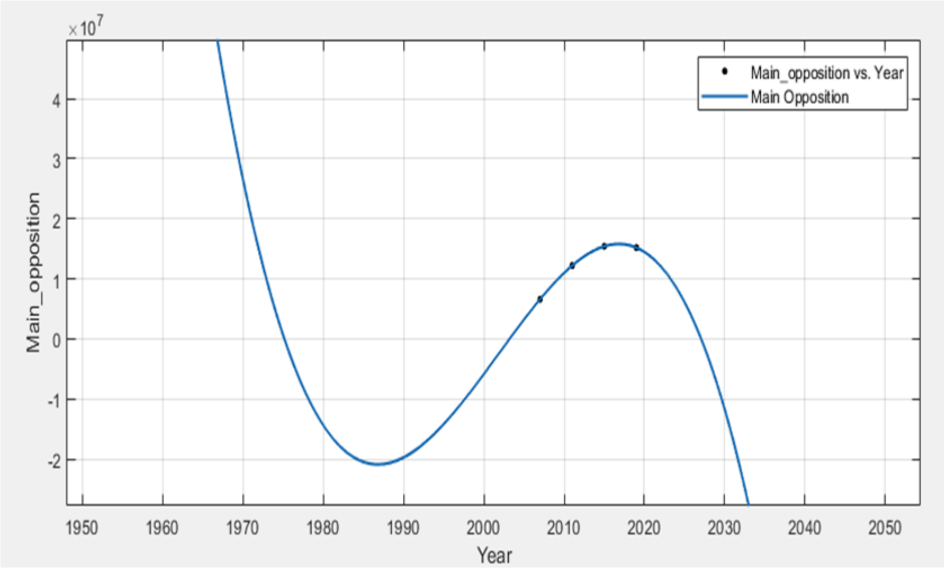

Figure 4. Fitted curve for major opposition party in presidential elections.

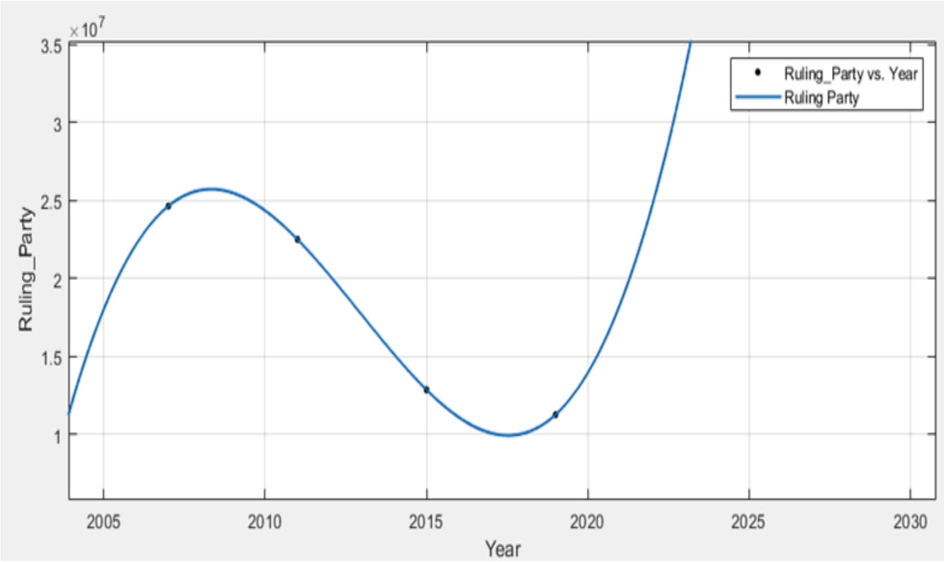

Figure 5. Fitted curve for the ruling party in presidential elections.

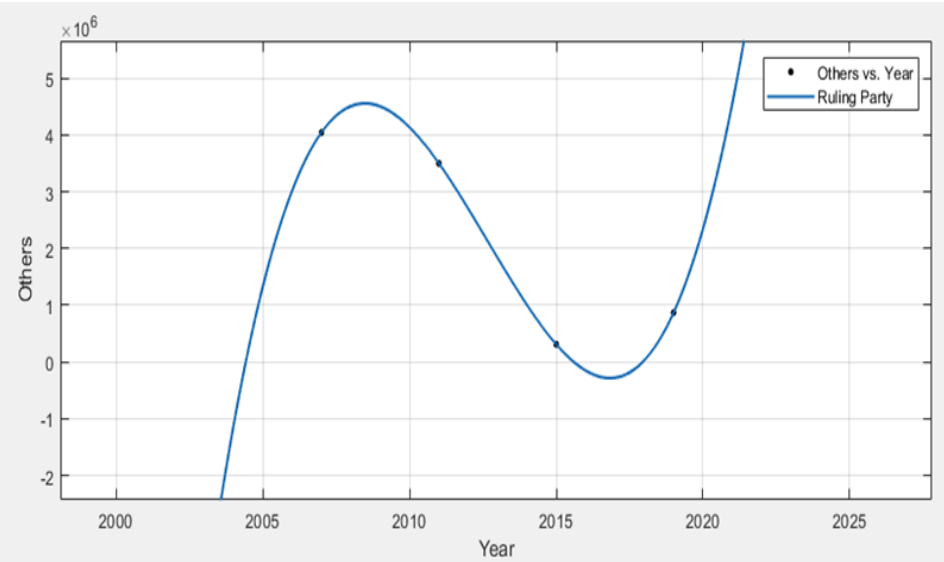

Figure 6. Fitted curve for minor opposition parties in presidential elections.

Table 7. Table of results for number of seats won at the senate by political parties.

Status	2007	2011	2015	2019	Ref
Ruling Party	85	71	48	45	INEC
Major opposition party	16	18	60	63	INEC
Minority opposition parties	8	20	1	1	INEC

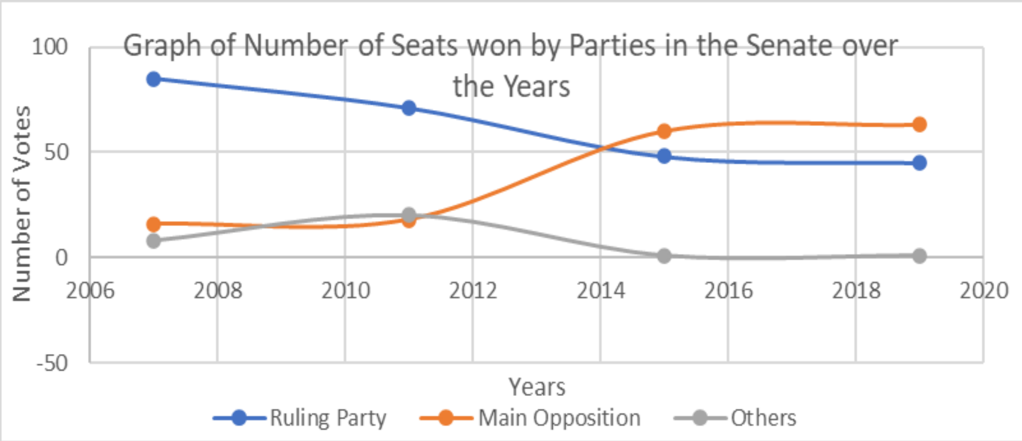

Figure 7. Comparative performance analysis of parties in senate election.

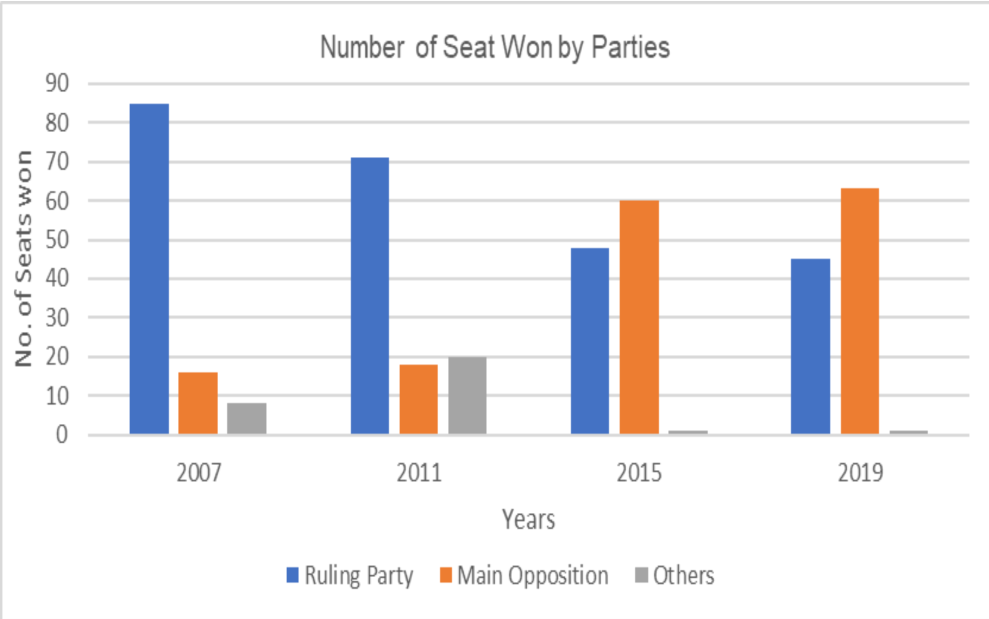

Figure 8. Bar chart representing political parties in senate elections.

-28.33, 0.4167, 58.47, and 38.94, respectively. The goodness of fit is also provided, with SSE of 1.551e -24 and R-square of 1. This implies that the predictions perfectly fit the data. The curve starts at a high point in 2007, with the major opposition party having a high number of seats in the Senate. However, the performance of the major opposition party declined significantly in 2008, with the number of seats decreasing sharply. The performance continued to decline in 2011, with the number of seats reaching a low point.

Figure 11 shows that the performance of minority opposition parties at the Senate has been fluctuating over the years. For instance, in 2019, the ruling party won 45 seats, the major opposition party won 63 seats, while minority opposition parties won only 1 seat. This suggests that the political landscape is dominated by the ruling party and the major opposition party, with minority opposition parties having limited influence. However, the performance of minority opposition parties can still be important in shaping the political

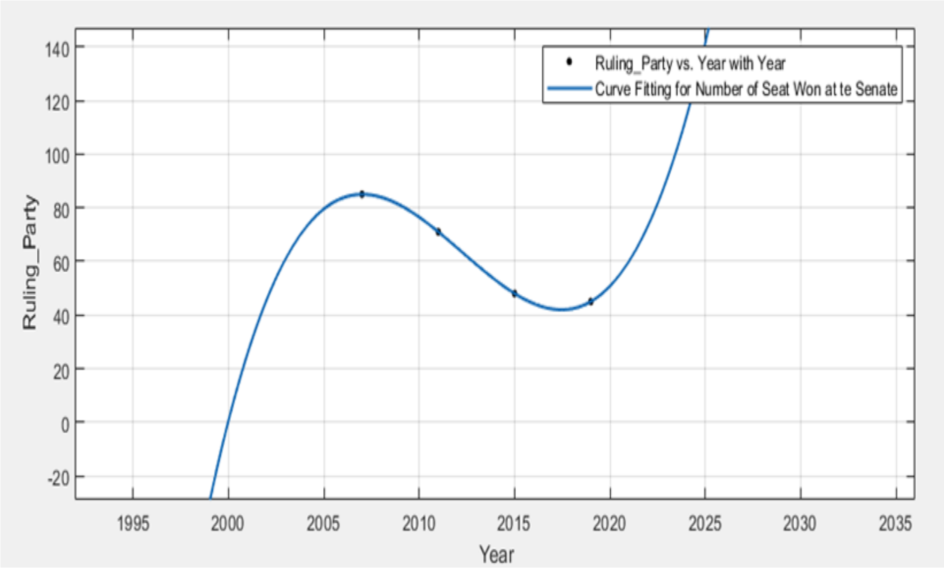

Figure 9. Fitted curve for the performance of the ruling party in senate elections.

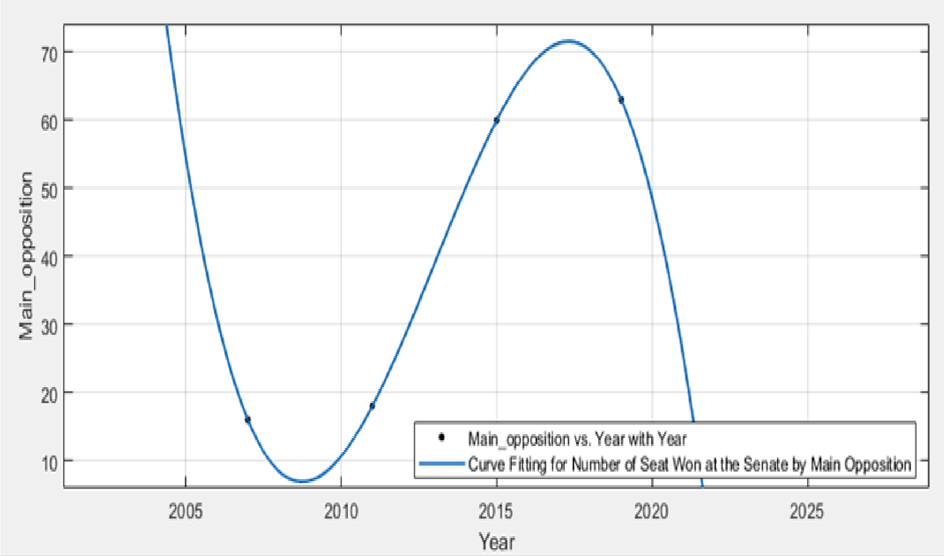

Figure 10. Fitted curve for the performance of the main opposition party in senate elections.

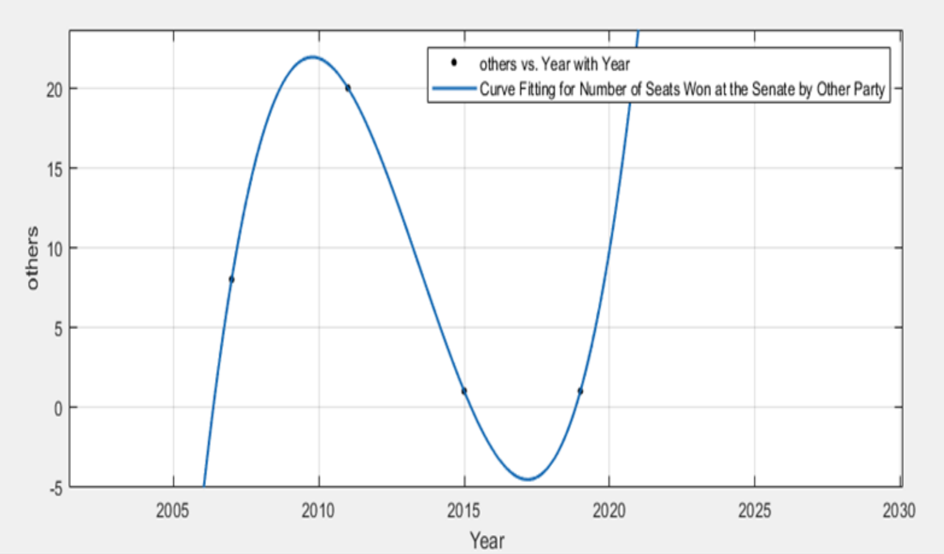

Figure 11. Fitted curve for the performance of the other parties in senate elections.

discourse and in providing alternative viewpoints.

3.4. Case 3-Analysis for Seats won by House of Representatives (HoR) over the Years

Figures 12 and 13 show that apart from the ruling party, the major opposition party has been the second-largest party at the HoR, with the number of seats won increasing over the years. The major opposition party won 62 seats in 2007, but 202 seats in 2019. This suggests that the major opposition party has been gaining support and influence over the years, and that it has been successful in challenging the dominance of the ruling party. Furthermore, minority opposition parties have had limited influence at the HoR, with the number of seats won fluctuating over the years. Minority opposition parties won 36 seats in 2007, but only 30 seats in 2019. This suggests that minority opposition parties have faced challenges in gaining support and influence, and that they have been overshadowed by the dominance of the ruling party and the growing influence of the major

Table 8. Table of results for number of seats won by political parties.

Status	2007	2011	2015	2019	Ref
Ruling Party	262	203	212	128	INEC
Major opposition party	62	69	140	202	INEC
Minority opposition parties	36	88	8	30	INEC

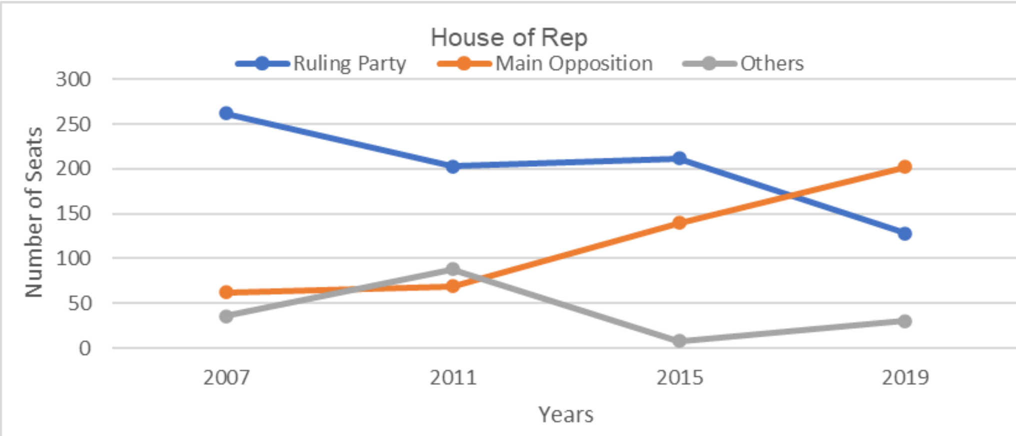

Figure 12. Comparative performance analysis of parties in HoR elections.

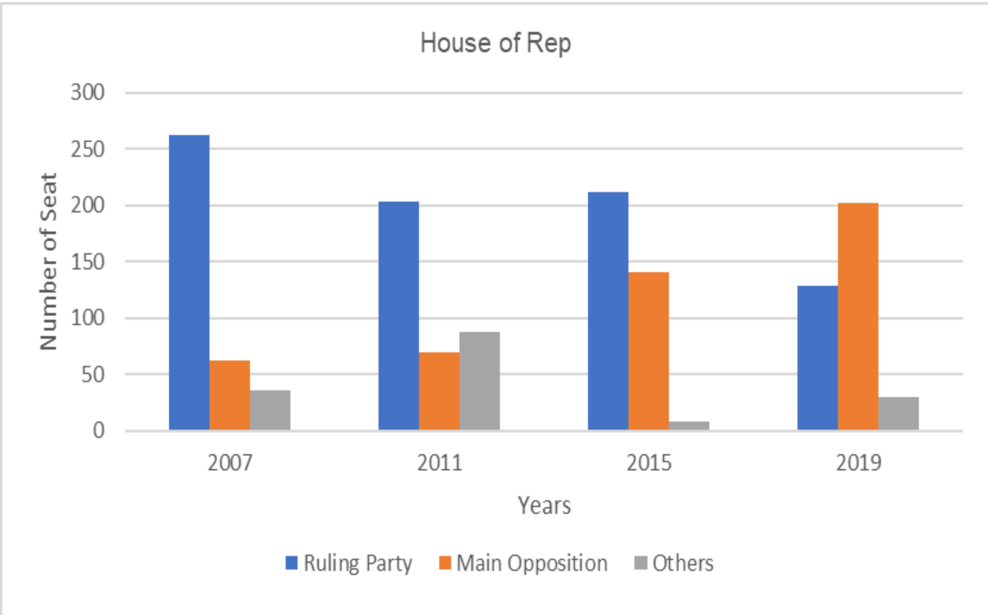

Figure 13. Bar chart representing political parties in HoR elections.

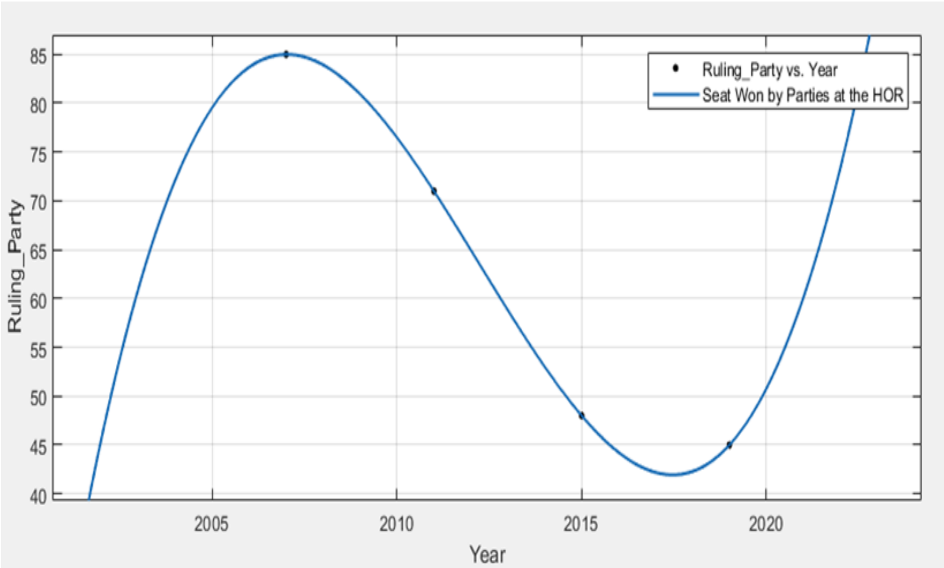

Figure 14. Fitted curve for ruling party in the HoR elections.

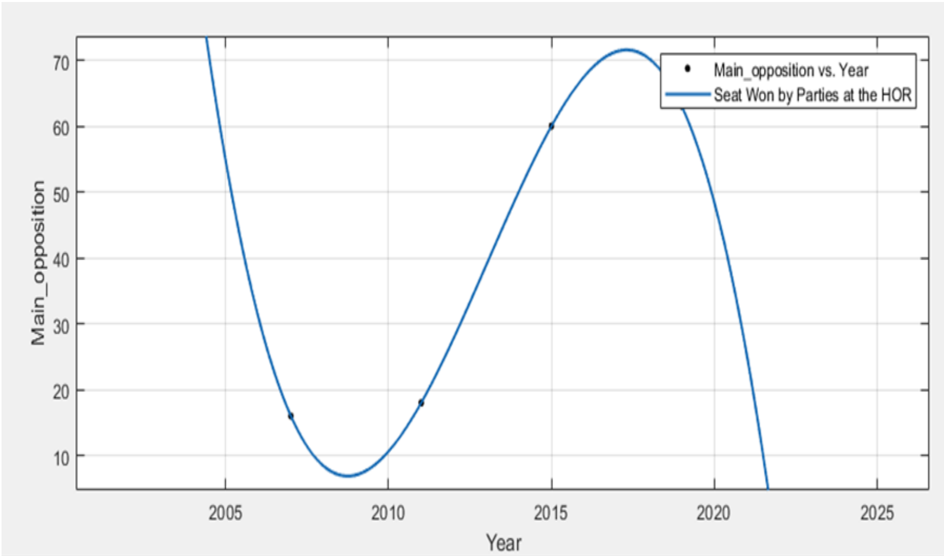

Figure 15. Fitted curve for major opposition party in the HoR elections.

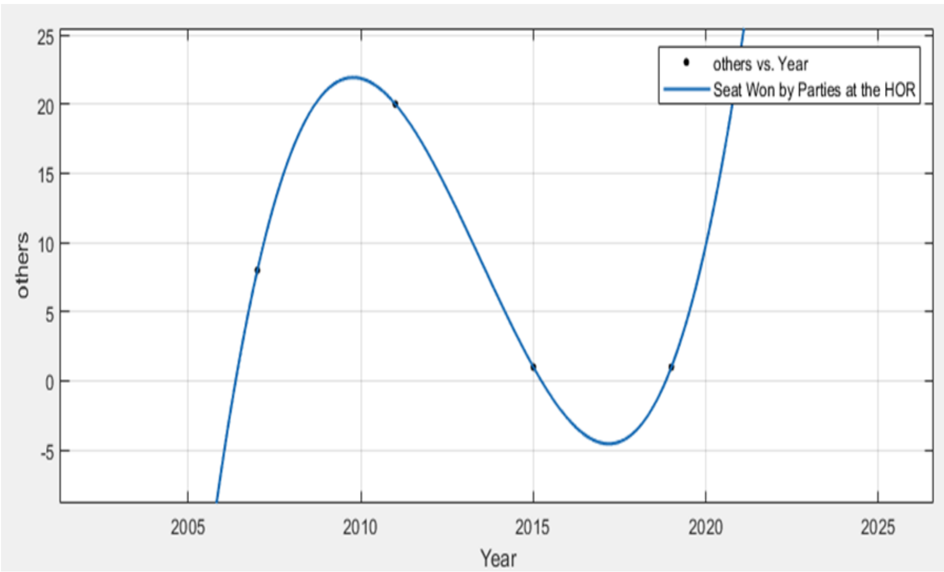

Figure 16. Fitted curve for minority opposition parties in the HoR elections.

Table 9. Table of results for number of state won by political parties.

Status	2007	2011	2015	2019	Ref
Ruling Party	24	23	10	13	INEC
Major opposition party	5	6	21	20	INEC
Minority opposition parties	7	7	5	3	INEC

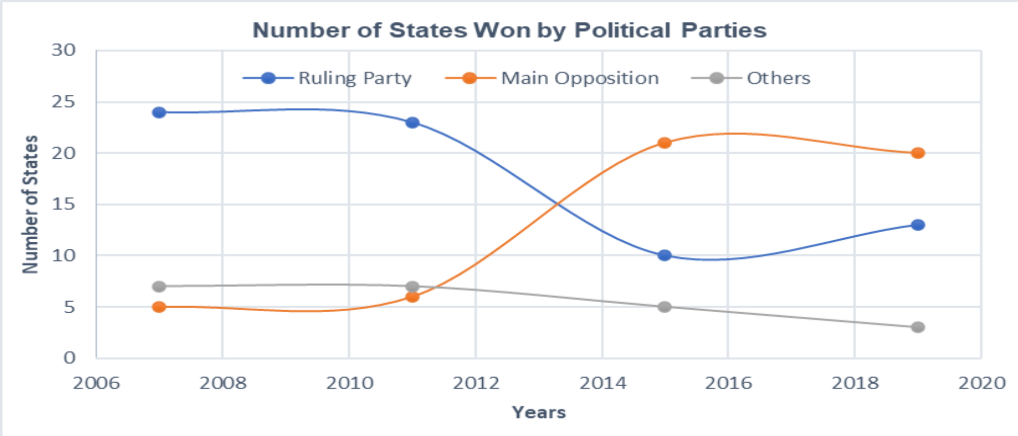

Figure 17. Performance plots of parties in governorship elections.

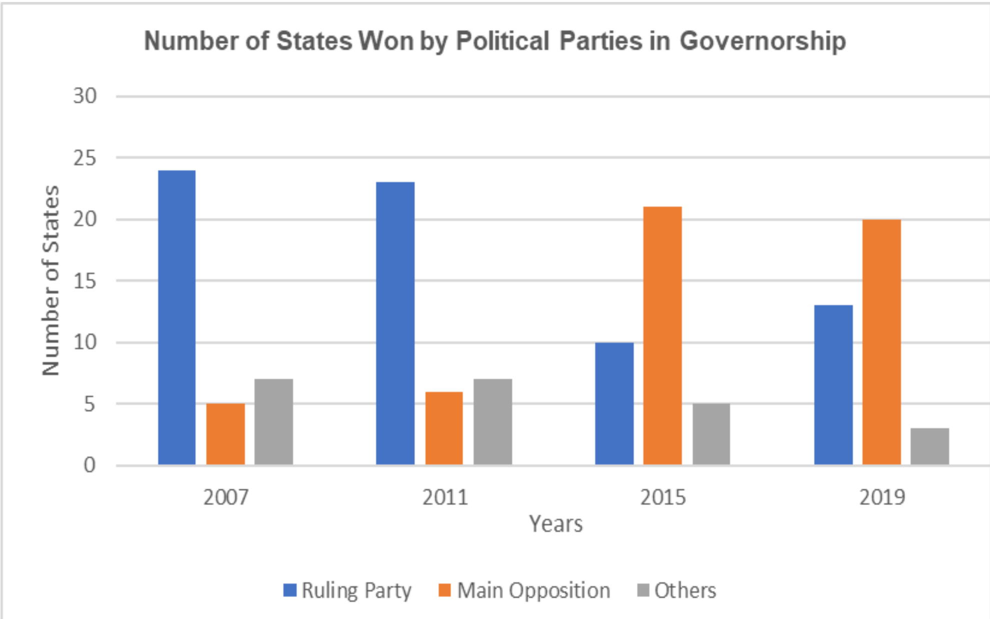

Figure 18. Bar chart for political parties in governorship elections.

opposition party.

Figure 14 represents the performance of the ruling party in the HoR which provides a useful tool for analysing the trend in the number of seats won by the ruling party over the years. The curve shows that the performance of the ruling party has been relatively stable, with the number of seats won fluctuating within a narrow range.

Figure 15 shows that the major opposition party has been successful in challenging the dominance of the ruling party at the HoR. The increase in the number of seats won by the major opposition party suggests that it has been gaining support and influence over the years. However, the linear model suggests that the performance of the major opposition party is affected by various factors such as changes in leadership, policy decisions, and public opinion. The linear model can be useful for predicting future trends in the performance of the major opposition party and for informing policy decisions.

Figure 16 represents the performance of minority

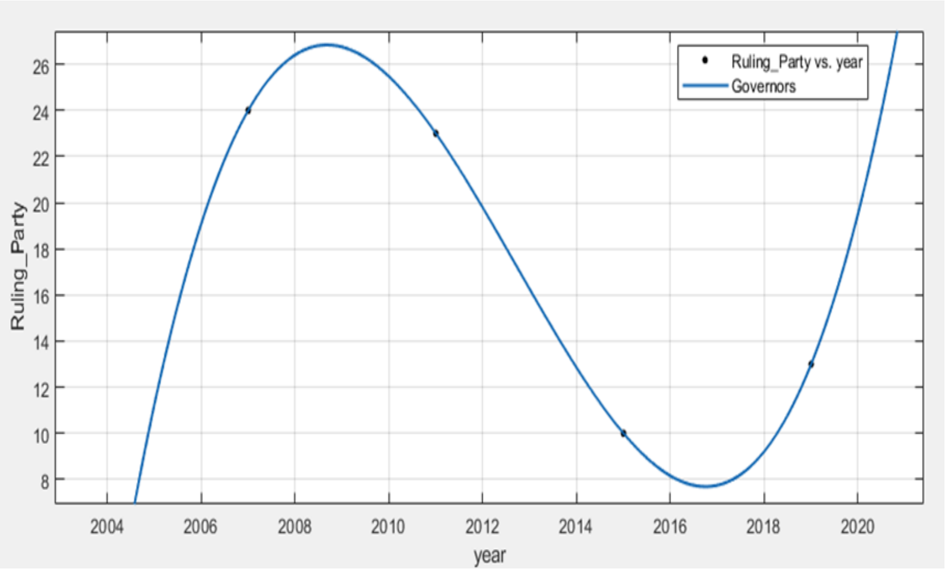

Figure 19. Fitted curve for the number of states won by the ruling party.

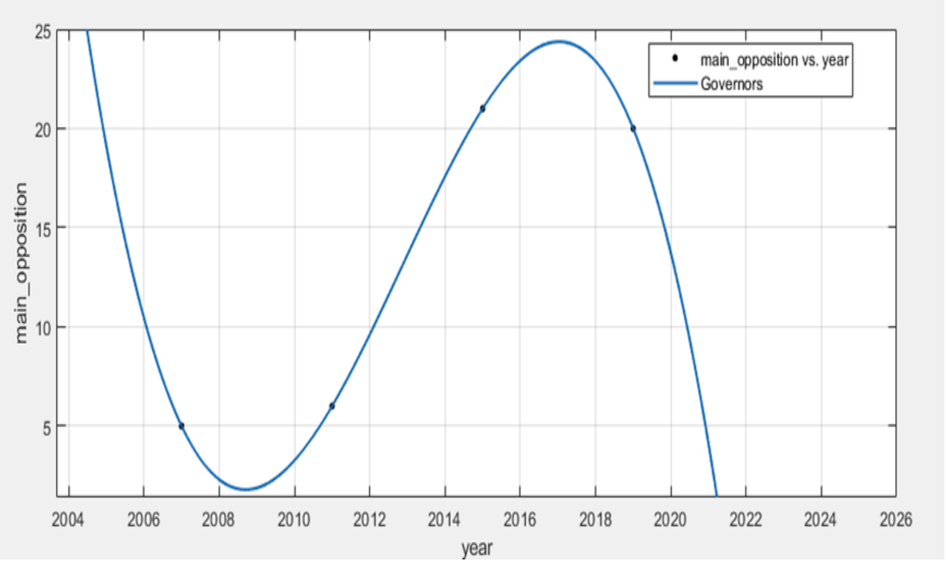

Figure 20. Fitted curve for the number of states won by the major opposition party.

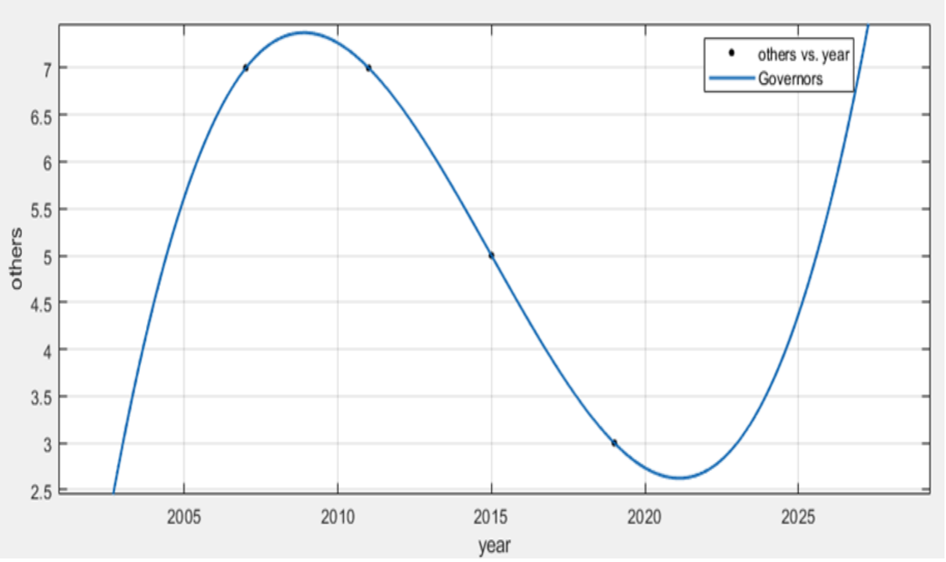

Figure 21. Fitted curve for the number of states won by the minor opposition parties.

opposition parties in the HoR. The curve provides valuable insights into the political landscape of the country. The increasing trend in the number of seats won by minority opposition parties from 2007 to 2011 indicates a growing opposition to the ruling party. However, the sharp decline in the number of seats won by minority opposition parties in 2015 suggests that the ruling party regained its dominance in the HoR. The slight increase in the number of seats won by minority opposition parties in 2019 indicates that the opposition is still present, but it is not strong enough to challenge the ruling party's dominance.

3.5. Case 4-Analysis of Governorship Elections over the Years

Figure 17 reflects the performance of the ruling party, major opposition party, and minority opposition parties in governorship elections from 2007 to 2019.

The bar chart in Figure 18 compares the performance of political parties in different elections. The chart clearly shows the changes in the number of states won by each party over time. The chart also shows that other parties have not been successful in winning states and may struggle to do so in future elections.

Importantly, the curve of Figure 19 shows that the ruling party's performance has been inconsistent, with a gradual increase in the number of states won from 2007 to 2008, followed by a sharp drop in 2011, and a gradual increase from 2011 to 2015. The curve also shows a sharp drop in the number of states won by the ruling party in 2015, followed by a slight increase in 2019.

In comparison to the fitted curve for the ruling party, the fitted curve for the major opposition party in Figure 20 shows a different trend. While the ruling party's performance has been inconsistent, the major opposition party's performance has been consistent, with a gradual increase in the number of states won over the years. This suggests that the major opposition party may be gaining ground in Nigerian politics and may be a viable alternative to the ruling party in future elections.

Figure 21 shows that the performance of minority opposition parties has been declining over the years and may continue to do so in future elections. However, the slight increase in the number of states won by minority opposition parties in 2015 suggests that minority opposition parties may still be able to bounce back in Nigerian politics.

3.6. Suggestion for Further Studies

Based on our model analysis and findings in this study, we suggest that further studies on this topic should incorporate the following recommendations:

a. elections malpractices like result manipulations, unfair electoral processes, vote buying, snatching of ballot boxes, unwholesome intimidation of some eligible voters, cancellation of election results, and lack of INEC transparency in the conduct of elections.
b. use of religious and tribal sentiment to woo eligible voters.
c. use of government machineries to frustrate opposition parties by the government in power use of judiciary and money inducement to truncate political mandates by political big wigs.

4. CONCLUSIONS

In this study, electoral processes are being modelled using a schematic diagram and set of non-linear differential equations that describe the dynamics of elections. The model has been used to predict possible patterns for electoral outcomes over the years for both the ruling parties, major opposition parties and minority opposition parties. Numerical simulation and curve fittings are used to identify sensitive parameters and data visualization. For each of the case studies, the fitted curve for the ruling party and the main opposition party shows a different trend. While the ruling party's performance has been inconsistent, the main opposition party's performance has been consistent, with a gradual increase in the number of states won over the years. This suggests that the main opposition party may be gaining ground in Nigerian politics and may be a viable alternative to the ruling party and vice versa to minority opposition parties in future elections. We are able to establish the fact that the outcome of an election is a function of voter's trust, candidate's popularity and policies of a political party. The model also proves that political party campaigners do not have long-term effect on deciding who wins in an election contest as their influence becomes weak and wane over time. Winning therefore becomes the decision of eligible voters and candidate's credibility. The choice an eligible voter makes is influenced by a number of factors, including membership of a party and the interest that the voter has developed in such party's ideologies. In addition, since democratic leadership is being subjected to electoral processes, voter's decision at the polling units becomes fundamental in predicting the outcome of elections. The reflection of this reality is that; Electoral Reform Act as amended, must be fully implemented. The use of political thugs and other electoral malpractices to win elections should be highly discouraged in order to avoid political agitations, insecurity in the country, lack of positive representatives at the parliaments and the economic hardship posed by such leadership. This menace in the lack of free and fair elections could be brought under control or eliminated by using the result of this research.

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Conflicts of Interest

The authors declare no conflict of interest.

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